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ABSTRACT

Detailed deterministic derivation of kinetic equations for relativistic plasmas is given. Focus is made on the dynamic of one-coordinate distribution functions of various tensor dimensions, but the closed set of kinetic equations is constructed of three functions: the scalar distribution function, the vector distribution function of dipole moment, and the vector distribution function of velocity (or the dipole moment in the momentum space). All two-coordinate distribution functions are discussed as well. They are presented together with their limits existing in the self-consistent field approximation. The dynamics of the small amplitude spin-electron-acoustic waves in the dense degenerate plasmas is studied within the kinetic model. This work presents the deterministic approach to the derivation and interpretation of the kinetic equations. So, no probability is introduced during the transition from the level of individual particles to the collective functions. The problem of thermalization is not considered, but we can see that the structure of kinetic equations is kept for the system before and after thermalization. Hence, the kinetic equations can be used to approach this item.

I. INTRODUCTION

Kinetic theory is one of the fundamental theories of the macroscopic phenomena in various physical systems.[1–8] Particularly, it plays an essential role in the plasma physics.[9] Kinetic theory requires proper microscopic derivation, where the evolution of the distribution function is traced from the evolution of individual particles. The distribution function shows the relative positions of all particles in the six-dimensional phase space and it can be consistently defined in the deterministic way. So, no probability theory is applied for the derivation or interpretation of kinetic equations or the distribution function. Such a view on the kinetic theory is the extension of the classical hydrodynamics based on the tracing of the microscopic dynamics of individual particles.[10,11]

Classical mechanics gives us dynamics of particles in the 3N-dimensional configurational space. The model of motion of particles is composed of the individual trajectories. Each of them, formally, takes place in its own three-dimensional space. This picture of motion is not obvious if we consider the Newtonian form of mechanics, where we can use our own imagination to place all particles in the single three-dimensional space. However, the Lagrangian and Hamiltonian forms of mechanics show the multidimensional structure of the theory more clearly. Moreover, the construction of the mathematical model out of trajectories (which are one-dimensional mathematical objects) distinguishes the classical mechanics from the hydrodynamics or the electrodynamics, which are the field theories.

The derivation of the hydrodynamics from the mechanics requires the representation of the mechanics in the form of the field theory. So, the mechanics would have the same mathematical structure as the electrodynamics, which is essentially important for the dynamics of charged particles. References 10 and 11 basically present the field form of classical mechanics, where the dynamics of particles takes the form of the dynamics of material fields. Inevitably, the field form of classical mechanics appears in the form of hydrodynamic equations. These are equations of nonequilibrium hydrodynamics that require further reduction for the particular forms of the collective motion. Particularly, it leads to some generalized form of nonequilibrium thermodynamics. Choosing a regime of thermal equilibrium allows us to get the first law of thermodynamics from the energy evolution equation. The possibility to consider different regimes, like regimes close to the thermal equilibrium and regimes out of equilibrium, shows that this approach can be used for the study of the thermalization process in the physical systems considering the dynamics of the systems in terms of the collective variables (material fields). It would require proper truncation of the hydrodynamics out equilibrium considering higher rank material fields to include the mechanism of relaxation in the model. However, it is an open problem that can be addressed within this formulation.

The formulation of the mechanics in terms of three-dimensional material fields leads to the hydrodynamics form of equations. While we consider the relative positions and relative momentums of all particles, we go to the distribution of particles in the six-dimensional space (the phase space of coordinate and momentum). This form of classical mechanics is considered in this paper. If we consider the microscopic evolution of point-like particles, we get the following distribution of the particle number in the coordinate space:[12]

$$n_m(\mathbf{r},t)=\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i(t)),\qquad (1)$$

where $\mathbf{r}_i(t)$ is the coordinate of the ith particle. This approach is suggested by Klimontovich.[1,12] While we use notation $n_m(\mathbf{r},t)$ on the left-hand side of Eq. (1), we should write $n_m(\mathbf{r},\mathbf{r}_1(t),\ldots,\mathbf{r}_N(t))$. So, the time dependence of function n_m is directly constructed out of the time evolution of coordinates of particles. The parameterization of the physical space is introduced here $\mathbf{r}$. So, this form gives us an intermediate step from the mechanics of trajectories to the field form of classical mechanics.

Considering the microscopic distribution (1) different authors applied some form of averaging (see, for instance, Refs. 1 and 13). Sometimes, the authors use some unspecified distributions $\langle n_m\rangle$ (see, for instance, Ref. 1). Otherwise, the authors use a specific form of averaging via some distribution function (see, for instance, Refs. 13 and 14). Anyways, the authors try to impose a probabilistic interpretation of the procedure of averaging. Let us mention that the aim of averaging is to make the transition to the macroscopic scale by averaging on the physically infinitesimal volume, but usually it is being replaced by the averaging on some distribution function.

Systematic, nonstatistical, definition of the averaging on the physically infinitesimal volume is suggested by Drofa and Kuz'menkov[10] (see also Refs. 15 and 16)

$$n_e(\mathbf{r},t)=\frac{1}{\Delta}\int_{\Delta}d\xi\sum_{i=1}^{N/2}\delta(\mathbf{r}+\boldsymbol{\xi}-\mathbf{r}_i(t)).\qquad (2)$$

Equation (2) contains vector ξ that scans the physically infinitesimal volume (it is illustrated in Fig. 1). However, the physically infinitesimal volume appears as the Δ-vicinity of each point of space. It is assumed that N is the full number of particles in the system. It is composed of two species: electrons with numbers $i\in[1,N/2]$ and protons $i\in[N/2+1,N]$. We need to trace the evolution of each species in the system. With no restrictions, we illustrate our derivation on the subsystem of electrons, so subindex e is dropped for all functions describing electrons in equations below $n_e\equiv n$.

A similar background is suggested for the kinetic theory. So, the microscopic distribution function is composed of delta functions in the physical coordinate space and the momentum space

$$f_m(\mathbf{r},\mathbf{p},t)=\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i(t))\delta(\mathbf{p}-\mathbf{p}_i(t)).\qquad (3)$$

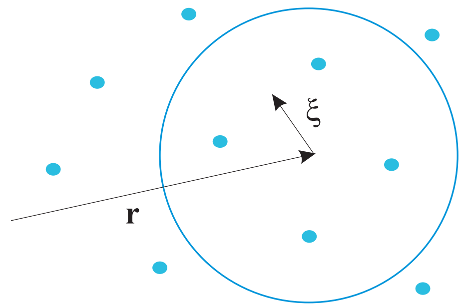

FIG. 1. The illustration of the Δ-vicinity in the coordinate space is given.

Transition to the macroscopic level is made similar to Eq. (2). We introduce the physically infinitesimal volume in a six-dimensional phase space

$$f(\mathbf{r},\mathbf{p},t)=\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p}d\xi d\boldsymbol{\eta}\sum_{i=1}^{N/2}\delta(\mathbf{r}+\boldsymbol{\xi}-\mathbf{r}_i(t))\delta(\mathbf{p}+\boldsymbol{\eta}-\mathbf{p}_i(t)),\qquad (4)$$

where $d\boldsymbol{\eta}$ is the element of volume in the momentum space, with $\int_{\Delta_p}d\boldsymbol{\eta}$ integral over Δ_p-vicinity in the momentum space, and $\Delta\equiv\Delta_r$ is the delta vicinity in the coordinate space. Recent development of this approach can be found in Refs. 17–20.

This discussion is about the field form of classical mechanics. However, similar insight is imposed on quantum mechanics. So, the formulation of the many-particle quantum mechanics in the form of evolution of the material fields is developed.[21–28]

The discussion presented above is focused on the fundamental background of hydrodynamics and kinetics, since the theoretical part of this paper is based on the development of the kinetic theory. However, the application of the kinetic theory to the high-density magnetized spin polarized degenerate electron gas is also considered. Particularly, we focus on the spin-electron-acoustic waves (SEAWs). Existence of the SEAWs in the partially spin-polarized degenerate plasmas was theoretically suggested in 2015.[29] They appear as the longitudinal waves of density and electric field, which propagate parallel to the anisotropy direction in the magnetized electron gas.[29,30]

If we have no spin polarization, we find a single longitudinal wave in the electron gas under assumption of the motionless ions. It is the Langmuir wave, which corresponds to the collective oscillations of electrons relative to the motionless ions. If we include the spin-polarization, we also find the SEAWs. Let us describe the physical picture. It corresponds to the relative motion of electrons with different spin projections. The spin polarization manifests itself in different partial number densities of electrons with the particular spin polarization. Hence, the relative oscillations of electrons with different amplitudes of the number density give the oscillation of a small proportion of electrons relative to motionless ions as well.[29,30] The SEAWs show similarity to the spin-plasmons considered in the two-dimensional structures.[31–40] Recently, the SEAWs have been considered in the degenerate electron gas,[41] while the background of the applied model is given in Refs. 16, 42 and 43.

The number density increase in electrons n_{0e} up to values, where the Fermi energy is proportional to the rest energy of electron $\varepsilon_{Fe}=(3\pi^2 n_{0e})^{2/3}\hbar^2/2m_e\sim m_e c^2$, gives a noticeable change in the dispersion dependence of the SEAWs. It also corresponds to the change of the dispersion dependence of the Langmuir waves. It shows the decrease in the relative frequency of the Langmuir wave and SEAW. Moreover, the relativistic regime shows the growth of the amplitude of the number density of the SEAWs relative to the amplitude of the Langmuir waves for the chosen value of the electric field in these waves.

This paper is organized as follows. In Sec. II, the derivation method of the kinetic equation from the microscopic motion is demonstrated and the general structure of the kinetic equation is derived. In Sec. III, we present an approximate kinetic model, where the contribution of physically infinitesimal volume is considered in a minimal regime called the monopole regime. In Sec. IV, the self-consistent field approximation of monopole regime is considered. In Sec. V, the multipole approximation in the relativistic kinetic equation is presented as the generalization of the monopole regime considered in Sec. III. In Sec. VI, the self-consistent field approximation of the multipole approximation is presented. In Sec. VII, the kinetic equation for the evolution of the dipole moment vector distribution function is obtained. In Sec. VIII, the kinetic equation for the distribution function of velocity is derived. In Sec. IX, a closed set of three kinetic equations is discussed. In Sec. X, the spin-electron-acoustic waves in the spin polarized electron gas of high density are considered. In Sec. XI, a brief summary of the obtained results is presented.

II. DERIVATION OF THE VLASOV EQUATION TRACING THE MICROSCOPIC MOTION OF PARTICLES

Analysis of many-particle systems is the analysis of motion of the large number of particles of order of 10^{23}. For the classical systems, it leads to 10^{23} second order connected differential equations. Nowadays, it is still impossible to use some computing powers to solve such problems. Moreover, it would be unimaginable to understand the obtained solution if we find one. We can expect that the majority of the physically relevant situations lead to some hierarchy of time, space, energy, and entropy scales. It means that the straightforward solution of the large number differential equations is meaningless. Kinetic theory is an example of the theoretical tools, where we can distinguish this hierarchy. The formation of different scales during the evolution of physical systems comes from the interaction between particles leading to the cause–effect sequences. In the kinetic theory, these sequences are hidden in the relation between different one-coordinate distribution functions, between one-coordinate and two-coordinate distribution functions, etc. On the other hand, we have the probabilistic approach to the many-particle systems. Let us speak on the classical systems since the quantum systems have two levels of description. It means that the wave function has the probabilistic interpretation, but the evolution of the wave function itself is determined by the nonstationary Schrödinger/Pauli equation. The probabilistic approach leads to "prediction" of the result of the many-particle evolution. Existing statistical theory does not provide blind prediction. It uses some information on the microscopic motion of individual particles and information on the form of their interaction. For instance, if we have a look at the derivation of the Vlasov equation in terms of BBGKY hierarchy, it starts with the Liouville equation obtained from the classical mechanics. It includes the interparticle interaction as a source of correlations. However, further integration on the ensemble of the similar physical systems with close (but different) initial conditions can disturb some physical correlations. It can also lead to the complexification of methods of analysis of the correlation function and following truncation technique. It can also lead to "cloudiness" in interpretation of hierarchy of time, space, energy, and other physical scales.

We need to consider the evolution of the distribution function. It requires the present equation of motion of each particle in full detail

$$\dot{\mathbf{p}}_i(t) = \mathbf{F}(\mathbf{r}_i(t), t), \tag{5}$$

where $\mathbf{p}_i(t) = m_i\mathbf{v}_i(t)/\sqrt{1-\mathbf{v}_i^2(t)/c^2}$ is the relativistic momentum of the ith particle, and $\mathbf{F}(\mathbf{r}_i(t), t)$ is the force acting on the ith particle from the electromagnetic field created by other particles in the system. The equation of motion for each particle appears as the evolution of the momentum under the Lorentz force action

$$\mathbf{F}(\mathbf{r}_i(t), t) = \left(q_i\mathbf{E}(\mathbf{r}_i(t), t) + \frac{1}{c}q_i[\mathbf{v}_i(t), \mathbf{B}(\mathbf{r}_i(t), t)]\right), \tag{6}$$

where $\mathbf{E}_i = \mathbf{E}_{i,ext} + \mathbf{E}_{i,int}$, $\mathbf{B}_i = \mathbf{B}_{i,ext} + \mathbf{B}_{i,int}$, $\mathbf{E}_i = \mathbf{E}(\mathbf{r}_i(t), t)$, and $\mathbf{B}_i = \mathbf{B}(\mathbf{r}_i(t), t)$, while the external fields are included along with the field of interaction of particles. The electric $\mathbf{E}_{i,int}$ and magnetic $\mathbf{B}_{i,int}$ fields caused by particles surrounding the ith particle are $\mathbf{E}_{i,int} = -\nabla_i\varphi(\mathbf{r}_i(t), t) - \frac{1}{c}\partial_t\mathbf{A}(\mathbf{r}_i(t), t)$ and $\mathbf{B}_{i,int} = \nabla_i \times \mathbf{A}(\mathbf{r}_i(t), t)$ with[44]

$$\varphi(\mathbf{r}_i(t), t) = \sum_{j\neq i} q_j \int \frac{\delta\left(t - t' - \frac{1}{c}|\mathbf{r}_i(t) - \mathbf{r}_j(t')|\right)}{|\mathbf{r}_i(t) - \mathbf{r}_j(t')|} dt', \tag{7}$$

and

$$\mathbf{A}(\mathbf{r}_i(t), t) = \sum_{j\neq i} q_j \int \frac{\delta\left(t - t' - \frac{1}{c}|\mathbf{r}_i(t) - \mathbf{r}_j(t')|\right)}{|\mathbf{r}_i(t) - \mathbf{r}_j(t')|}\frac{\mathbf{v}_j(t')}{c} dt'. \tag{8}$$

Equations (7) and (8) allow to introduce the Green function of the retarding electromagnetic interaction[44]

$$\widetilde{G}_{ij} = \frac{\delta\left(t - t' - \frac{1}{c}|\mathbf{r}_i(t) - \mathbf{r}_j(t')|\right)}{|\mathbf{r}_i(t) - \mathbf{r}_j(t')|}. \tag{9}$$

We consider the high-temperature plasmas, where the electrons (and maybe other species) have the relativistic temperatures comparable with the rest energy of the electron m_ec^2 (of the particle of corresponding species). Obtaining such huge temperatures of the species would lead to a high degree of ionization of the atomic objects. Moreover, the two-particle interactions (collision-like processes) of the high-energy electrons with the ions would change the electron configurations of ions or provide additional ionization. These dynamical processes should contribute to the model. To avoid these complexifications, we consider the hydrogen plasmas, where we have electrons and protons only.

To find the evolution equation for the distribution function (4), we consider the time derivative of this function to obtain the following equation:

$$\partial_t f(\mathbf{r}, \mathbf{p}, t) + \nabla\cdot\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\boldsymbol{\xi} d\boldsymbol{\eta}\sum_{i=1}^{N/2}\dot{\mathbf{r}}_i(t)\delta(\mathbf{r}+\boldsymbol{\xi}-\mathbf{r}_i(t)) \times \delta(\mathbf{p}+\boldsymbol{\eta}-\mathbf{p}_i(t)) + \nabla_{\mathbf{p}}\cdot\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\boldsymbol{\xi} d\boldsymbol{\eta}\sum_{i=1}^{N/2}\dot{\mathbf{p}}_i(t) \times \delta(\mathbf{r}+\boldsymbol{\xi}-\mathbf{r}_i(t))\delta(\mathbf{p}+\boldsymbol{\eta}-\mathbf{p}_i(t)) = 0. \tag{10}$$

The second (third) term in this equation appears as the result of action of the time derivative on the delta-function containing the coordinate (the momentum).

The second term in Eq. (10) contains the following function:

$$\langle \mathbf{v}_i(t)\rangle \equiv \frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta} \sum_{i=1}^{N/2} \mathbf{v}_i(t)\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}, \tag{11}$$

where $\mathbf{v}_i(t) = d\mathbf{r}_i(t)/dt$, $\delta_{\mathbf{r}i} \equiv \delta(\mathbf{r}+\xi-\mathbf{r}_i(t))$, and $\delta_{\mathbf{p}i} \equiv \delta(\mathbf{p}+\boldsymbol{\eta}-\mathbf{p}_i(t))$. For the further representation of this function, we apply the relativistic expression for the velocity of a particle via its momentum $\mathbf{v}_i(t) = \mathbf{p}_i(t)c/\sqrt{\mathbf{p}_i^2(t)+m_i^2c^2}$, where the additional replacement of the momentum $\mathbf{p}_i(t)$ on $\mathbf{p}+\boldsymbol{\eta}$ can be applied. It gives the representation of function $\langle \mathbf{v}_i(t)\rangle$:

$$\langle \mathbf{v}_i(t)\rangle = \frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta} \sum_{i=1}^{N/2} \frac{(\mathbf{p}+\boldsymbol{\eta})c}{\sqrt{(\mathbf{p}+\boldsymbol{\eta})^2+m_i^2c^2}}\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}. \tag{12}$$

We need to extract term $\mathbf{v}\cdot f$, which is traditional for the physical kinetics. Hence, expression (12) is represented in required form as

$$\langle \mathbf{v}_i(t)\rangle = \mathbf{v}\cdot f(\mathbf{r},\mathbf{p},t) + \widetilde{\mathbf{F}}(\mathbf{r},\mathbf{p},t), \tag{13}$$

where

$$\widetilde{\mathbf{F}}(\mathbf{r},\mathbf{p},t) \equiv \langle \Delta\mathbf{v}_i(t)\rangle \tag{14}$$

with

$$\langle \Delta\mathbf{v}_i\rangle \equiv \frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta} \sum_{i=1}^{N/2}\left(\frac{\boldsymbol{\eta}c}{\sqrt{\mathbf{p}^2+m_i^2c^2}} - \frac{\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\eta})c}{\left(\sqrt{\mathbf{p}^2+m_i^2c^2}\right)^3}\right)\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}, \tag{15}$$

with $\mathbf{p} = m_s\mathbf{v}/\sqrt{1-\mathbf{v}^2/c^2}$ and $\mathbf{v} = \mathbf{p}c/\sqrt{\mathbf{p}^2+m_s^2c^2}$. In addition to the arguments of the vector distribution function $\widetilde{\mathbf{F}}(\mathbf{r},\mathbf{p},t)$, we use symbol "tilde" to distinguish it from the force acting on the individual particles (5).

We include the representation (13) in Eq. (10). We also include equations of motion of the individual particles (5)–(8) in the last term in Eq. (10):

$$\begin{aligned}
&\partial_t f(\mathbf{r},\mathbf{p},t) + (\mathbf{v}\cdot\nabla)f(\mathbf{r},\mathbf{p},t) + \nabla\cdot\widetilde{\mathbf{F}}(\mathbf{r},\mathbf{p},t) \\
&+\frac{q_s}{m_s}\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta}\sum_{i=1}^{N/2}\Bigg[\mathbf{E}_{ext}(\mathbf{r}+\xi,t) + \frac{1}{c}\bigg[(\mathbf{v}+\Delta\mathbf{v}_i) \\
&\times\mathbf{B}_{ext}(\mathbf{r}+\xi,t)\bigg] + q_{s'}\sum_{j=1,j\neq i}^{N}\int dt'\bigg(-\nabla_{\mathbf{r}} - \frac{1}{c}\frac{\mathbf{v}_j(t')}{c}\hat{\partial}_t \\
&+\frac{1}{c^2}\left[\mathbf{v}_i(t)\times\left[\nabla_{\mathbf{r}}\times\mathbf{v}_j(t')\right]\right]\bigg)G(\mathbf{r}+\xi-\mathbf{r}_j(t))\Bigg]\delta_{\mathbf{r}i}\cdot\nabla_{\mathbf{p}}\delta_{\mathbf{p}i} = 0,
\end{aligned} \tag{16}$$

where we use symbol $\hat{\partial}_t$ instead of the time derivative due to the change of the structure of argument of the Green function G. We replaced $\mathbf{r}_i(t)$ by $\mathbf{r}+\xi$ in the arguments of the external electric field, the external magnetic field, and the Green function using the delta-function $\delta_{\mathbf{r}i}$. Initially the action of the time derivative has the following form:

$$\partial_t G(t,t',\mathbf{r}_i(t),\mathbf{r}_j(t')) = \frac{\delta'}{|\mathbf{r}_i(t)-\mathbf{r}_j(t')|}, \tag{17}$$

where δ' is the derivative of the delta function on its argument. It is also can be rewritten in the following form:

$$\partial_t G(t,t',\mathbf{r},\mathbf{r}') = \frac{\delta'}{|\mathbf{r}+\xi-\mathbf{r}'-\xi'|}. \tag{18}$$

For the introduction of ξ', see the following equations.

Next, we use representation (13) in terms, which describes the interaction

$$\begin{aligned}
&\partial_t f + \mathbf{v}\cdot\nabla f + \nabla\cdot\widetilde{\mathbf{F}} + \frac{q_s}{m_s}\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta}\sum_{i=1}^{N/2}\bigg(\mathbf{E}(\mathbf{r}+\xi,t) \\
&+\frac{1}{c}[(\mathbf{v}+\Delta\mathbf{v}_i)\times\mathbf{B}(\mathbf{r}+\xi,t)]\bigg)\delta_{\mathbf{r}i}\cdot\nabla_{\mathbf{p}}\delta_{\mathbf{p}i} \\
&+\frac{q_s}{m_s}q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int_{\Delta,\Delta_p}\frac{d\xi d\boldsymbol{\eta} d\xi' d\boldsymbol{\eta}'}{\Delta^2\Delta_p^2}\sum_{i=1}^{N/2}\sum_{j=1,j\neq i}^{N}\int dt' \\
&\times\left(-\nabla_{\mathbf{r}} - \frac{1}{c}\frac{(\mathbf{v}'+\Delta\mathbf{v}_j')}{c}\partial_t - \frac{1}{c^2}\left[(\mathbf{v}+\Delta\mathbf{v}_i)\times\left[(\mathbf{v}'+\Delta\mathbf{v}_j')\times\nabla_{\mathbf{r}}\right]\right]\right) \\
&\times G(\mathbf{r}+\xi-\mathbf{r}'-\xi')\delta_{\mathbf{r}i}\cdot\nabla_{\mathbf{p}}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j} = 0,
\end{aligned} \tag{19}$$

where $\delta_{\mathbf{r}'j} \equiv \delta(\mathbf{r}'+\xi'-\mathbf{r}_j(t))$ and $\delta_{\mathbf{p}'j} \equiv \delta(\mathbf{p}'+\boldsymbol{\eta}'-\mathbf{p}_j(t))$.

The presented kinetic equation (19) contains the deviation of coordinates ξ from the center of the Δ-vicinity $\mathbf{r}$. It also contains the deviation of the velocity $\Delta\mathbf{v}_i$ from the value corresponding to the center of the $\Delta_{\mathbf{p}}$-vicinity in the momentum space $\mathbf{v} = \mathbf{p}c/\sqrt{\mathbf{p}^2+m_s^2c^2}$. This is the intermediate general representation of the kinetic equation, which is considered below under some additional assumptions.

III. MONOPOLE APPROXIMATION OF THE KINETIC EQUATION FOR THE SCALAR DISTRIBUTION FUNCTION

To consider the monopole approximation of the kinetic equation, we need to neglect the contribution of ξ, ξ', $\boldsymbol{\eta}$ (including $\Delta\mathbf{v}_i$), and $\boldsymbol{\eta}'$ (including $\Delta\mathbf{v}_j'$) in the dynamical functions. However, on this step, we present the kinetic equation, where the monopole approximation is considered for the space variables only. Hence, we neglect the contribution of ξ, and ξ', but we keep the contribution of $\boldsymbol{\eta}$,

$$\begin{aligned}
&\partial_t f + \mathbf{v}\cdot\nabla f + \nabla\cdot\widetilde{\mathbf{F}} + \frac{q_s}{m_s}\left(\mathbf{E}(\mathbf{r},t) + \frac{1}{c}[\mathbf{v}\times\mathbf{B}(\mathbf{r},t)]\right)\cdot\nabla_{\mathbf{p}} f \\
&+\frac{q_s}{m_s c}\left(\nabla_{\mathbf{p}}\cdot\left[\widetilde{\mathbf{F}}(\mathbf{r},\mathbf{p},t)\times\mathbf{B}(\mathbf{r},t)\right]\right) + \frac{q_s}{m_s}q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int_{\Delta,\Delta_p}\frac{d\xi d\boldsymbol{\eta} d\xi' d\boldsymbol{\eta}'}{\Delta^2\Delta_p^2} \\
&\times\sum_{i=1}^{N/2}\sum_{j=1,j\neq i}^{N}\int dt'\left(-\nabla_{\mathbf{r}} - \frac{\mathbf{v}'}{c^2}\partial_t + \frac{1}{c^2}[\mathbf{v}\times[\nabla\times\mathbf{v}']]\right)G(\mathbf{r},\mathbf{r}')\delta_{\mathbf{r}i} \\
&\cdot\nabla_{\mathbf{p}}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j} = 0.
\end{aligned} \tag{20}$$

Transition from Eq. (19) to Eq. (20) includes the transformation like $\mathbf{E}(\mathbf{r}+\xi,t) \approx \mathbf{E}(\mathbf{r},t)$ and

$$\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p} d\xi d\boldsymbol{\eta}\sum_{i=1}^{N/2}\mathbf{E}(\mathbf{r},t)\delta_{\mathbf{r}i}\cdot\nabla_{\mathbf{p}}\delta_{\mathbf{p}i} = \mathbf{E}(\mathbf{r},t)\cdot\nabla_{\mathbf{p}} f, \tag{21}$$

where the electric field $\mathbf{E}(\mathbf{r}, t)$ is placed outside of the integral, while the rest is the derivative of the distribution function on the momentum.

Equation (20) allows to introduce the two-particle (or in other terms two-coordinate) distribution functions. To the best of my knowledge, the majority of papers and books, including my own works, use the notion "two-particle distribution functions." It can be confusing to some extent since the model describes the many-particle systems. Hence, the distribution function and the two-particle distribution function actually describe the dynamics of whole the system. Sometimes, the two-particle distribution functions are called two-coordinate distribution functions. It looks more logical since it contains two coordinates $\mathbf{r}$ and $\mathbf{r}'$ (or more exactly two coordinates in the phase space $\{\mathbf{r}, \mathbf{p}\}$ and $\{\mathbf{r}', \mathbf{p}'\}$).

Let us present the following kinetic equation in the monopole approximation both on the coordinate and the momentum (we do it due to the technical reasons, since we do not want to introduce several two-coordinate distribution functions, but we show the complete picture below):

$$\partial_t f + \mathbf{v}\cdot\nabla f + \nabla\cdot\widetilde{\mathbf{F}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\cdot\nabla_{\mathbf{p}} f + \frac{q_s}{m_s c}(\nabla_{\mathbf{p}}\cdot[\widetilde{\mathbf{F}}\times\mathbf{B}]) + \frac{q_s}{m_s}q_{s'}\int d\mathbf{r}' d\mathbf{p}'\int dt' \times\left(-\nabla_{\mathbf{r}} - \frac{\mathbf{v}'}{c^2}\partial_t + \frac{1}{c^2}[\mathbf{v}\times[\nabla\times\mathbf{v}']]\right)G(\mathbf{r},\mathbf{r}')\cdot\nabla_{\mathbf{p}} f_2 = 0, \quad (22)$$

where $q_{s'} f_2 = q_e f_{2,ee} + q_i f_{2,ei}$ and

$$f_{2,ee}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') = \int_{\Delta,\Delta_p}\frac{d\xi d\boldsymbol{\eta} d\xi' d\boldsymbol{\eta}'}{\Delta^2\Delta_p^2}\sum_{i=1}^{N/2}\sum_{j=1,j\neq i}^{N/2}\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j} \equiv \langle\langle 1\rangle\rangle \quad (23)$$

is the two-coordinate distribution function. The compact notation is also introduced for the two-coordinate distribution function constructed of double brackets $\langle\langle 1\rangle\rangle$. We use this notation below for other two-coordinate distribution functions.

Let us discuss the condition of neglecting ξ and ξ' at the transition to Eqs. (20) and (21). In Eq. (19), we consider the electric field $\mathbf{E}(\mathbf{r}_i, t)$ and other functions on the scale of the Δ-vicinity. The coordinate of the ith particle is replaced with the combination $\mathbf{r} + \boldsymbol{\xi}$, where $\mathbf{r}$ is the center of the vicinity, while vector $\boldsymbol{\xi}$ scans the vicinity. Here, vector $\boldsymbol{\xi}$ has particular value $\boldsymbol{\xi}_i$ for the ith particle (if it is inside the vicinity). The possibility of the expansion on parameter ξ depends on the change of the electric field on the scale of the vicinity. If we consider the propagation of the high-frequency/short-wavelength radiation through the plasmas, we have the fast nonmonotonic change of the electric field over the vicinity at $\lambda < \Delta^{1/3}$. Therefore, the expansion is not allowed. In the opposite regime $\lambda > \Delta^{1/3}$ (it would be even better for $\lambda > 10\Delta^{1/3}$), the change of the electric field can be considered as relatively small. Below, it is discussed that $\Delta^{1/3} \sim r_{De}$. Hence, the value of Δ and the condition on the wavelength depend on the concentration. We cannot expand on $\boldsymbol{\xi}$ if the system is under action of the x-rays, but we can expand on $\boldsymbol{\xi}$ if the system is under action of the radio frequency radiation. If we consider the interparticle interaction, presented by terms containing the Green function G, we see two parameters ξ and ξ'. Parameter ξ is included in the dependence of the resulting electromagnetic field on the coordinate. So, it is equivalent to the situation described above for the external field. Parameter ξ' is similar to ξ, but it is related to the particles creating field, while parameter ξ is related to particles under the action of this field. We have less control of the inner field. However, the magnetic field depends on the velocities of particles, so we can estimate it by the estimation of the velocities of flows and the thermal velocities. The radiation of particles depends on the acceleration of particles during interaction, but it can be roughly estimated via the temperature using the Planck distribution. The estimation of temperature of the system and velocity of flows gives some information on the inner fields. It allows us to estimate the part of the inner field, which can be modeled using the suggested approximation of expansion on ξ.

IV. THE SELF-CONSISTENT FIELD APPROXIMATION IN CLASSIC MONOPOLE KINETICS FOR THE EQUATION OF EVOLUTION OF SCALAR DISTRIBUTION FUNCTION

In order to consider the self-consistent field (the mean-field) approximation, we need to split the two-coordinate distribution function $f_{2,ee}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t')$ into the product of two one-coordinate distribution functions $f_{2,ee}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') = f(\mathbf{r},\mathbf{p},t)\cdot f(\mathbf{r}',\mathbf{p}',t')$. Hence, the term containing the interaction in Eq. (22) reappears in the following form:

$$\frac{q_s}{m_s}q_{s'}\nabla_{\mathbf{p}} f(\mathbf{r},\mathbf{p},t)\cdot\Bigg(-\nabla_{\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt' G(\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t') - \frac{1}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\mathbf{v}'\partial_t G(\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t') + \frac{1}{c^2}\left[\mathbf{v}\times\left[\nabla\times\int d\mathbf{r}'d\mathbf{p}'\int dt'\mathbf{v}'\right]\right]\Bigg)G(\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t') = 0. \quad (24)$$

It allows us to introduce the scalar and vector potentials of the self-consistent electromagnetic field

$$\varphi_{int}(\mathbf{r},t) = \int d\mathbf{r}'d\mathbf{p}'\int dt' G(\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t') \quad (25)$$

and

$$\mathbf{A}_{int}(\mathbf{r},t) = \frac{1}{c}\int d\mathbf{r}'d\mathbf{p}'\int dt'\mathbf{v}' G(\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t'). \quad (26)$$

Some discussion on the self-consistent field approximation for the kinetics based on the deterministic microscopic motion of particles is given in Ref. 45 for the nonrelativistic kinetics and hydrodynamics.

So, we find the well-known form of the Vlasov kinetic equation for the systems of relativistic particles[46]

$$\partial_t f + \mathbf{v}\cdot\nabla f + q_s\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial\mathbf{p}} = 0, \quad (27)$$

where $\mathbf{E} = \mathbf{E}_{ext} + \mathbf{E}_{int}$, $\mathbf{B} = \mathbf{B}_{ext} + \mathbf{B}_{int}$, the electric field is caused by the distribution of charges in the coordinate space: $\mathbf{E}_{int} = -\nabla\varphi - \partial_t\mathbf{A}_{int}/c$, $\mathbf{B}_{int} = \nabla\times\mathbf{A}_{int}$. Equation (27) appears to be coupled with the electromagnetic field: $\nabla\times\mathbf{E}_{int} = -\partial_t\mathbf{B}_{int}/c$, $\nabla\cdot\mathbf{B}_{int} = 0$,

$$\nabla\times\mathbf{B}_{int} = \partial_t\mathbf{E}_{int}/c + (4\pi/c)\sum_s q_s\int\mathbf{v}f_s(\mathbf{r},\mathbf{p},t)d\mathbf{p}, \quad (28)$$

and

$$\nabla \cdot \mathbf{E}_{int} = 4\pi \sum_s q_s \int f_s(\mathbf{r},\mathbf{p},t) d\mathbf{p}. \quad (29)$$

The self-consistent/macroscopic/mean field is introduced in this section. It corresponds to the possibility of the multiplication of the two-coordinate distribution function on the product of two one-coordinate distribution functions. However, this statement is rather formal. Let us specify that it means in terms of presented method. Noncorrelated part of the two-coordinate distribution function corresponds to the distances between centers of two vicinities larger than the diameter of the vicinity. In the opposite case, we have an overlapping of the vicinities which is related to the interparticle interaction at the small distances. This part of the interaction also includes that no self-action of the particle exists. So, the condition $i \neq j$ in the summation in the definition of the two-coordinate distribution function explicitly works for the correlation. The uncorrelated/mean field part corresponds to the interaction of two groups of particles, since these two groups composed of different sets of particles.

Our goal is to derive the kinetic theory from the motion of individual particles. It is a transition from a large number of discrete trajectories to the evolution of the continuous distribution functions. However, if one deals with the kinetic equations in order to model physical processes, he needs to apply some numerical methods. The particle-in-cell (PIC) method is one of highly important methods for the numerical solving the Vlasov–Maxwell equations. Let us discuss a canonical symplectic PIC method for solving the Vlasov–Maxwell equations by discretizing its canonical Poisson bracket following work.[47] The distribution function f is discretized in the phase space through the Klimontovich representation. It is made by the introduction of a finite number of the Lagrangian sampling points, which play the role of the effective macroscopic particles. The electromagnetic field is also discretized. Hence, the Hamiltonian functional is expressed as a function of the sampling points and the discretized electromagnetic field. It leads to a finite-dimensional Hamiltonian system with a canonical symplectic structure. The total number of degrees of freedom for the constructed effective system is composed as $D = 3N + 3M$, where N denotes the total number of the Lagrangian sampling points and M denotes the total number of discrete grid-points. This method is particularly interesting for our research due to two items. First, this method goes to the discretization of the kinetic formalism as all numerical methods do. For us, it is interesting how this discretization is made. This method makes a step back from the continuous distribution functions to a set of a finite number of the Lagrangian sampling points considered as effective particles. Second, we can consider it from the microscopic point of view, instead of a method of solution of the macroscopic equations. This method constructs a system of effective "quasiparticles" on the macroscopic scale. So, it introduces an effective space averaging method in order to replace almost infinite number of real particles to a finite number of the Lagrangian sampling points. Moreover, the discretization of the electromagnetic field includes a particular feature described in Ref. 47. The discretization itself is given by Eq. (8) and below. The shift of particles from the grid points is discussed in Ref. 48 before Eq. (14). It is mentioned in Ref. 48 since it is necessary to get the potentials in the positions of particles, while almost all particles are shifted from grids. The method of calculation of the off-grid values of potentials is presented in Ref. 48 by the interpolation techniques. In our approach, we can find a similarity to the introduction of the dipole-distribution functions.

V. MULTIPOLE APPROXIMATION IN THE RELATIVISTIC KINETIC EQUATION FOR THE SCALAR DISTRIBUTION FUNCTION (THE VLASOV EQUATION)

A. The interaction of particles with the external electromagnetic field

The multipole expansion of the electric field $\mathbf{E}_{ext}(\mathbf{r}+\xi,t)$, the magnetic field $\mathbf{B}_{ext}(\mathbf{r}+\xi,t)$, and the Green function $G(\mathbf{r}-\mathbf{r}'+\xi-\boldsymbol{\xi}',t-t')$ leads to the appearance of new distribution functions of different tensor ranks. Let us introduce these functions before we find their appearance from the expansion of Eq. (19) on ξ and ξ' up to the second order on these vectors.

First, we introduce the vector distribution function of the electric dipole moment (divided by the charge q_s) or the displacement of particles relative to the center of the Δ-vicinity,

$$d^a(\mathbf{r},\mathbf{p},t) = \langle \xi^a \rangle = \int_{\Delta,\Delta_p} \frac{d\boldsymbol{\xi} d\boldsymbol{\eta}}{\Delta\Delta_p} \sum_{i=1}^{N} \xi^a \delta_{\mathbf{r}i}\delta_{\mathbf{p}i}. \quad (30)$$

We also include in our analysis the distribution function of the electric quadrupole moment (divided by the charge q_s)

$$Q^{ab}(\mathbf{r},\mathbf{p},t) = \langle \xi^a\xi^b \rangle = \int_{\Delta,\Delta_p} \frac{d\boldsymbol{\xi} d\boldsymbol{\eta}}{\Delta\Delta_p} \sum_{i=1}^{N} \xi^a\xi^b \delta_{\mathbf{r}i}\delta_{\mathbf{p}i}. \quad (31)$$

The presented functions contain vector $\boldsymbol{\xi}$ scanning the Δ-vicinity. Functions (30) and (31) appear as two examples of the infinite set of the distribution functions containing different degrees of vector ξ (the product of different numbers of projections to construct an element of the tensor of corresponding rank).

As it is demonstrated by Eqs. (11)–(14), we can find the velocity of the particle under the integral defining the distribution function. We can also find the product of several projections of the velocity (of the same particle, in order to get the one-coordinate distribution function) or the product of the projection of the velocity on the projections of vector ξ.

One of such distribution functions appearing in our derivation is the distribution function of flux of the electric dipole moment (divided by the charge q_s)

$$J_D^{ab}(\mathbf{r},\mathbf{p},t) = \langle v_i^a(t)\xi^b \rangle = \frac{1}{\Delta^2}\frac{1}{\Delta_p^2}\int_{\Delta,\Delta_p} d\boldsymbol{\xi} d\boldsymbol{\eta} d\boldsymbol{\xi}' d\boldsymbol{\eta}' \sum_{i,j=1,j\neq i}^{N} v_i^a(t)\xi^b \delta_{\mathbf{r}i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (32)$$

It is more useful to extract the velocity corresponding to the center of $\Delta_{\mathbf{p}}$-vicinity in the momentum space, like we make it for $\langle v_i^a(t)\rangle$ in Eqs. (11)–(14). Therefore, function (32) leads to the following distribution function:

$$j_D^{ab}(\mathbf{r},\mathbf{p},t) = \langle \Delta v_i^a(t)\xi^b \rangle = \frac{1}{\Delta^2}\frac{1}{\Delta_p^2}\int_{\Delta,\Delta_p} d\boldsymbol{\xi} d\boldsymbol{\eta} d\boldsymbol{\xi}' d\boldsymbol{\eta}' \sum_{i,j=1,j\neq i}^{N} (\Delta v_i^a(t))\xi^b \delta_{\mathbf{r}i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (33)$$

Let us introduce one more distribution function, which is the third rank tensor. It is the distribution function of flux of the electric quadrupole moment (divided by the charge q_s)

$$J_Q^{abc}(\mathbf{r},\mathbf{p},t) = \langle v_i^a(t)\xi^b\xi^c\rangle = \frac{1}{\Delta^2}\frac{1}{\Delta_p^2}\int_{\Delta,\Delta_p} d\boldsymbol{\xi} d\boldsymbol{\eta} d\boldsymbol{\xi}' d\boldsymbol{\eta}' \sum_{i,j=1,j\neq i}^{N} v_i^a(t)\xi^b\xi^c\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (34)$$

For this distribution function, we can also introduce the reduced flux of the electric quadrupole moment on the velocities in the local frame $\Delta v_i^a(t)$: $j_Q^{abc}(\mathbf{r},\mathbf{p},t) = \langle \Delta v_i^a(t)\xi^b\xi^c\rangle$.

Functions (30)–(34) appear in the Vlasov-like kinetic equation for the scalar distribution function. First, these functions appear at the analysis of the action of the external electromagnetic field on the system of charged particles. Here, we need to consider the fourth term in Eq. (19). We apply the expansion of the external electromagnetic field on the deviations of coordinate $\boldsymbol{\xi}$ from the center of the Δ-vicinity of point $\mathbf{r}$. This expansion requires the slow change of the fields $\mathbf{E}$ and $\mathbf{B}$ over the diameter $\sqrt[3]{\Delta}$ of the Δ-vicinity. Particularly, if we consider the electromagnetic wave, we assume that the wavelength λ is larger than the diameter $\sqrt[3]{\Delta}$: $\lambda \geq \sqrt[3]{\Delta}$.

For instance, if we consider diameter $\sqrt[3]{\Delta} \approx 0.1\ \mu$m, we get an estimation on the minimal value of the frequency of the electromagnetic wave of order of $\omega_{max} \sim 10^{16}$ s^{-1}. So, we can consider electromagnetic waves with frequencies up to the frequencies of the visible light, but the ultraviolet radiation is out of the range of applicability of the model. If we deal with the dense medium, we can choose the delta-vicinity of the smaller value. So, the range of the ultraviolet radiation can be included in our analysis.

During the derivation of the kinetic equations, we consider the expansion up to the second order on ξ^a: $\mathbf{E}(\mathbf{r}+\boldsymbol{\xi},t) = \mathbf{E}(\mathbf{r},t) + \xi_a\partial_\mathbf{r}^a\mathbf{E}(\mathbf{r},t) + (1/2)\xi_a\xi_b\partial_\mathbf{r}^a\partial_\mathbf{r}^b\mathbf{E}(\mathbf{r},t)+\cdots$ and $\mathbf{B}(\mathbf{r}+\boldsymbol{\xi},t) = \mathbf{B}(\mathbf{r},t) + \xi_a\partial_\mathbf{r}^a\mathbf{B}(\mathbf{r},t) + (1/2)\xi_a\xi_b\partial_\mathbf{r}^a\partial_\mathbf{r}^b\mathbf{B}(\mathbf{r},t)+\cdots$. Hence, the fourth term in Eq. (19) can be rewritten as

$$\Omega_0 = \frac{q_s}{m_s}\langle(\mathbf{E}_{ext}(\mathbf{r}+\boldsymbol{\xi},t) + \frac{1}{c}[\mathbf{v}\times\mathbf{B}_{ext}(\mathbf{r}+\boldsymbol{\xi},t)] + \frac{1}{c}[\Delta\mathbf{v}_i\times\mathbf{B}_{ext}(\mathbf{r}+\boldsymbol{\xi},t)])\nabla_\mathbf{p}\rangle \quad (35)$$

before the expansion. After the described expansion, we obtain

$$\Omega_0 = \frac{q_s}{m_s}\Big\{\mathbf{E}_{ext}(\mathbf{r},t)\nabla_\mathbf{p}f(\mathbf{r},\mathbf{p},t) + (\partial_\mathbf{r}^a\mathbf{E}_{ext})\nabla_\mathbf{p}d^a(\mathbf{r},\mathbf{p},t) + \frac{1}{2}(\partial_\mathbf{r}^a\partial_\mathbf{r}^b\mathbf{E}_{ext})\nabla_\mathbf{p}Q^{ab}(\mathbf{r},\mathbf{p},t) + \frac{1}{c}[\mathbf{v}\times\mathbf{B}_{ext}(\mathbf{r},t)]\nabla_\mathbf{p}f(\mathbf{r},\mathbf{p},t) + \frac{1}{c}[\mathbf{v}\times(\partial_\mathbf{r}^a\mathbf{B}_{ext})]\nabla_\mathbf{p}d^a(\mathbf{r},\mathbf{p},t) + \frac{1}{c}\Big[\mathbf{v}\times(\partial_\mathbf{r}^a\partial_\mathbf{r}^b\mathbf{B}_{ext})\Big]\nabla_\mathbf{p}Q^{ab}(\mathbf{r},\mathbf{p},t) + \frac{1}{c}\varepsilon^{abc}\Big[B_{ext}^b\nabla_\mathbf{p}^cF^a(\mathbf{r},\mathbf{p},t) + (\partial_\mathbf{r}^dB_{ext}^b)\nabla_\mathbf{p}^cj^{ad}(\mathbf{r},\mathbf{p},t) + (\partial_\mathbf{r}^d\partial_\mathbf{r}^fB_{ext}^b)\nabla_\mathbf{p}^cj^{adf}(\mathbf{r},\mathbf{p},t)\Big]\Big\}. \quad (36)$$

To specify the novelty of our work, let us give the following illustration. We consider the mean-field approximations meaning that we neglect the short-range correlations associated with collisions. These correlations are usually represented as the collisional integrals. We show that the dynamics of plasmas in these approximations leads to appearance of the vector and tensor distribution functions associated with the nonsymmetric distributions of charged particles on the scale of the physically infinitesimal volume. This microscopic dynamics gives an effect on the macroscopic dynamics as well. As a qualitative parallel, we can refer to the motion in a rapidly oscillating field, where "smooth" motion of a particle is affected by dynamics on the small time scale (see Ref. 49, Sec. 30). More direct comparison exists in the standard electrodynamics, where we find the dipole moment, the quadrupole moment, etc., if we consider the distribution of charges and introduce characteristic of the whole system.

The appearance of the vector distribution functions is not necessarily related to the microscopic dynamics. If we include the spin of particles, it leads to the spin distribution function, which is a vector function describing collective evolution of spins of particles (see Refs. 17, 30, and 50–52). However, the form and consequences of the spin contribution is different. Moreover, these two approaches can be considered simultaneously.

B. The interparticle interaction

Let us introduce function Ω, which presents a term in the kinetic equation describing the interparticle interaction [the last term in Eq. (19)]. We split this term on three parts $\Omega = \Omega_1 + \Omega_2 + \Omega_3$, where are $\Omega_1 = -(q_s/m_s)\langle\nabla_i\varphi_{i,int}\cdot\nabla_{p,i}\rangle$, $\Omega_2 = -(q_s/m_sc)\langle\partial_t\mathbf{A}_{i,int}\cdot\nabla_{p,i}\rangle$, and $\Omega_3 = (q_s/m_sc)\varepsilon^{abc}\varepsilon^{cfg}\langle v_i^b(t)\partial_i^f A_{i,int}^g\cdot\nabla_{p,i}\rangle$, and we use short notation for the average on the physically infinitesimal volume, like

$$\langle\nabla_i\varphi_{i,int}\cdot\nabla_{p,i}\rangle = \int_{\Delta_\mathbf{r}\Delta_\mathbf{p}}\frac{d\boldsymbol{\xi}d\boldsymbol{\eta}}{\Delta_\mathbf{r}\Delta_\mathbf{p}}\sum_i\nabla_i\varphi_{i,int}\delta_{\mathbf{r}i}\cdot\nabla_{p,i}\delta_{\mathbf{p}i}. \quad (37)$$

Here, functions $\varphi_{i,int}$ and $\mathbf{A}_{i,int}$ are given by Eqs. (7) and (8).

Let us consider the multipole expansion for each of function Ω_j ($j = 1, 2, 3$). We start this presentation with Ω_1 which can be represented in the following form:

$$\Omega_1 = -\int\frac{d\mathbf{r}'d\mathbf{p}'}{(\Delta_\mathbf{r}\Delta_\mathbf{p})^2}\int dt'\int_{\Delta_\mathbf{r}\Delta_\mathbf{p}}d\boldsymbol{\xi}d\boldsymbol{\eta}\int_{\Delta_{\mathbf{r}'}\Delta_{\mathbf{p}'}}d\boldsymbol{\xi}'d\boldsymbol{\eta}' \times\sum_{i,j,j\neq i}\frac{q_iq_j}{m_s}\nabla_\mathbf{r}\big(G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}')\big)\delta_{\mathbf{r}i}\cdot\nabla_{p,i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (38)$$

We make the expansion of the Green functions on $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$. We also expand the velocity of particle v_i^b on the deviation from the average velocity $\boldsymbol{\eta}$. For the Green function $G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}')$, we have $G = G_0 + \xi^bG_{1,b} + (1/2)\xi^b\xi^cG_{2,bc}$, where $G_0 = \delta(t-t'-|\mathbf{r}-\mathbf{r}'|/c)/|\mathbf{r}-\mathbf{r}'|$, $G_{1,b} = \partial_\mathbf{r}^bG_0$, and $G_{2,bc} = \partial_\mathbf{r}^b\partial_\mathbf{r}^cG_0$.

Function Ω_1 can be represented in terms of two-coordinate distribution functions:

$$\Omega_1 = -\frac{q_sq_{s'}}{m_s}\nabla_\mathbf{p}\int d\mathbf{r}'d\mathbf{p}'\int dt'\Big[(\nabla_\mathbf{r}G_0)f_{2,ss'}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') + (\nabla_\mathbf{r}G_1^\beta)\Big(d_{2,ss'}^b(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') - d_{2,ss'}^b(\mathbf{r}',\mathbf{r},\mathbf{p},\mathbf{p}',t,t')\Big) + \frac{1}{2}(\nabla_\mathbf{r}G_2^{bc})\Big(q_{2,ss'}^{bc}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') + q_{2,ss'}^{bc}(\mathbf{r}',\mathbf{r},\mathbf{p},\mathbf{p}',t,t') - D_{2,ss'}^{bc}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') - D_{2,ss'}^{bc}(\mathbf{r}',\mathbf{r},\mathbf{p},\mathbf{p}',t,t')\Big)\Big], \quad (39)$$

where we use the following notations for the two-coordinate distribution functions:

$$d^b_{2,ss'}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') \equiv \langle\langle \xi^b \rangle\rangle \to d^b_s(\mathbf{r},\mathbf{p},t)\cdot f_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad (40)$$

$$q^{bc}_{2,ss'}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') \equiv \langle\langle \xi^b\xi^c \rangle\rangle \to q^{bc}_s(\mathbf{r},\mathbf{p},t)\cdot f_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad (41)$$

and

$$D^{bc}_{2,ss'}(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') \equiv \langle\langle \xi^b\xi'^c \rangle\rangle \to d^b_s(\mathbf{r},\mathbf{p},t)\cdot d^c_{s'}(\mathbf{r}',\mathbf{p}',t'). \quad (42)$$

The potential part of the electric force $q_s\mathbf{E}$ of the interparticle interaction is presented in terms of the two-coordinate distribution function. Next, we make the same representation for the part of the electric force $q_s\mathbf{E}$ expressed via the vector potential. Therefore, let us demonstrate the explicit form of Ω_2 as follows:

$$\Omega_2 = -\int\frac{d\mathbf{r}'d\mathbf{p}'}{(\Delta_\mathbf{r}\Delta_\mathbf{p})^2}\int dt'\int_{\Delta_\mathbf{r}\Delta_\mathbf{p}} d\boldsymbol{\xi}d\boldsymbol{\eta}\int_{\Delta_{\mathbf{r}'}\Delta_{\mathbf{p}'}} d\boldsymbol{\xi}'d\boldsymbol{\eta}' \times\sum_{i,j,j\neq i}\frac{q_iq_j}{m_sc^2}\partial_t\left(\mathbf{v}_j(t')G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}')\right)\delta_{\mathbf{r}i}\cdot\nabla_{p,i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (43)$$

Function Ω_2 can be approximately represented in terms of two-coordinate distribution functions after expansion on ξ and ξ' as follows:

$$\Omega_2 = -\frac{1}{c^2}\frac{q_sq_{s'}}{m_s}\partial^a_\mathbf{p}\int d\mathbf{r}'d\mathbf{p}'\int dt'\Big[(\partial_tG_0)\langle\langle v^a_i(t)\rangle\rangle + (\partial_tG^b_1)\Big(\langle\langle\xi^bv^a_j(t')\rangle\rangle - \langle\langle\xi'^bv^a_j(t')\rangle\rangle\Big) + \frac{1}{2}(\partial_tG^{bc}_2) \times\Big(\langle\langle\xi^b\xi^cv^a_j(t')\rangle\rangle + \langle\langle\xi'^b\xi'^cv^a_j(t')\rangle\rangle - 2\langle\langle\xi^b\xi'^cv^a_j(t')\rangle\rangle\Big)\Big], \quad (44)$$

where we use several two-coordinate functions, which are presented via their correlationless limit in Appendix B [see Eqs. (B1)–(B8)].

Finally, we present the explicit form of the magnetic part of the interparticle interaction,

$$\Omega_3 = \frac{1}{c^2}\varepsilon^{abc}\varepsilon^{cfg}\frac{1}{(\Delta_\mathbf{r}\Delta_\mathbf{p})^2}\int d\mathbf{r}'d\mathbf{p}'\int dt' \times\int_{\Delta_\mathbf{r}\Delta_\mathbf{p}} d\boldsymbol{\xi}d\boldsymbol{\eta}\int_{\Delta_{\mathbf{r}'}\Delta_{\mathbf{p}'}} d\boldsymbol{\xi}'d\boldsymbol{\eta}'\sum_{i,j,j\neq i}\frac{q_iq_j}{m_s}v^b_i(t)v^g_j(t') \times\partial^f_\mathbf{r}\left(G(\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}')\right)\delta_{\mathbf{r}i}\cdot\partial^a_{p,i}\delta_{\mathbf{p}i}\delta_{\mathbf{r}'j}\delta_{\mathbf{p}'j}. \quad (45)$$

Here, we make the expansion on $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$ and make the interpretation of found terms via the two-coordinate distribution functions:

$$\Omega_3 = \frac{1}{c^2}\frac{q_sq_{s'}}{m_s}\varepsilon^{abc}\varepsilon^{cfg}\partial^a_\mathbf{p}\int d\mathbf{r}'d\mathbf{p}'\int dt'\Big[(\partial^f_\mathbf{r}G_0)\langle\langle v^b_i(t)v^g_j(t')\rangle\rangle + (\partial^f_\mathbf{r}G^k_1)\Big(\langle\langle\xi^kv^b_i(t)v^g_j(t')\rangle\rangle - \langle\langle\xi'^kv^b_i(t)v^g_j(t')\rangle\rangle\Big) + \frac{1}{2}(\partial^f_\mathbf{r}G^{kl}_2)\Big(\langle\langle\xi^k\xi^lv^b_i(t)v^g_j(t')\rangle\rangle + \langle\langle\xi'^k\xi'^lv^b_i(t)v^g_j(t')\rangle\rangle - 2\langle\langle\xi^k\xi'^lv^b_i(t)v^g_j(t')\rangle\rangle\Big)\Big], \quad (46)$$

where we use no specific notations for the two-coordinate distribution functions since we have many two-coordinate distribution functions. We have no reason to use special symbols for each of them. We also have following limits for these functions in the self-consistent field approximation, which are presented via their correlationless limit in Appendix B [see Eqs. (B9)–(B14)].

To conclude this section, we point out that we made the generalization of Eq. (22) up to the account of the quadrupole moment distribution function in the multipole expansion while Eq. (22) includes the distribution of charger (and currents as well) in the monopole approximation. The presented generalization is demonstrated by four terms labeled as Ω_0, Ω_1, Ω_2, and Ω_3. We consider the expansion on parameters ξ and η. Formally, we have functions containing the increasing degrees of these parameters. It leads to the convergent condition of the expansion. Formally, we can state that there is a small parameter, which allows the convergent of the expansion. Its existence shows that we consider the small vicinity presenting a macroscopically infinitesimally small area. The higher degrees of ξ appear in the distribution function, but they appear as the expansion of the electromagnetic field on ξ. So, we need to keep small the second dimensionless parameter $\lambda > \Delta^{1/3} \sim r_{De}$, which is discussed after Eq. (23).

VI. SELF-CONSISTENT FIELD APPROXIMATION IN THE MULTIPOLE APPROXIMATION FOR THE SCALAR DISTRIBUTION FUNCTION EVOLUTION EQUATION

We consider the dynamics of charged particles. Therefore, the mean-field or the self-consistent field approximation gives the major contribution in the dynamics of systems. Hence, we consider all two-coordinate distribution functions as the product of the corresponding one-coordinate distribution functions. Therefore, we combine Eqs. (36), (39), (44), and (46) represented in the self-consistent field approximation and find the following equation for the scalar distribution function:

$$\partial_tf_s + \mathbf{v}\cdot\nabla f_s + \nabla\cdot\widetilde{\mathbf{F}}_s + q_s\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_s}{\partial\mathbf{p}} + q_s\left(\partial^b\mathbf{E}+\frac{1}{c}\mathbf{v}\times\partial^b\mathbf{B}\right)\cdot\frac{\partial d^b_s}{\partial\mathbf{p}} + q_s\left(\partial^b\partial^c\mathbf{E}+\frac{q_s}{c}\mathbf{v}\times\partial^b\partial^c\mathbf{B}\right) \cdot\frac{\partial Q_s}{\partial\mathbf{p}} + \frac{q_s}{c}\varepsilon^{abc}\left(B^c\cdot\partial_{a,\mathbf{p}}\widetilde{F}^b_s + \partial^dB^c\cdot\partial_{a,\mathbf{p}}j^{bd}_{D,s} + \partial^d\partial^fB^c\cdot\partial_{a,\mathbf{p}}j^{bdf}_{Q,s}\right) = 0, \quad (47)$$

where the velocity is extracted from functions j^a_s, $j^{ab}_{D,s}$, and $j^{abc}_{Q,s}$. We also have $\mathbf{E} = \mathbf{E}_{ext} + \mathbf{E}_{int}$, and $\mathbf{B} = \mathbf{B}_{ext} + \mathbf{B}_{int}$.

The integral terms presenting the sources of the electromagnetic field are written as $\mathbf{E}_{int}$ and $\mathbf{B}_{int}$, while these functions obey the Maxwell equations in accordance with the explicitly found integral expressions for the electromagnetic field $\{\mathbf{E}_{int}, \mathbf{B}_{int}\}$ via the one-coordinate distribution functions:

$$\nabla\cdot\mathbf{B}_{int} = 0, \quad \nabla\times\mathbf{E}_{int} = -\frac{1}{c}\partial_t\mathbf{B}_{int}, \quad (48)$$

$$(\nabla\times\mathbf{B}_{int})^a = \frac{1}{c}\partial_tE^a_{int} + \frac{4\pi}{c}\sum_s q_s\left(\int j^a_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \partial^b\int J^{ab}_{s,D}(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \frac{1}{2}\partial^b\partial^c\int J^{abc}_{s,Q}(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \cdots\right), \quad (49)$$

$$\nabla\cdot\mathbf{E}_{int}=4\pi\sum_s q_s\left(\int f_s(\mathbf{r},\mathbf{p},t)d\mathbf{p}+\partial^b\int d_s^b(\mathbf{r},\mathbf{p},t)d\mathbf{p}+\frac{1}{2}\partial^b\partial^c\int Q_s^{bc}(\mathbf{r},\mathbf{p},t)d\mathbf{p}+\cdots\right). \tag{50}$$

The self-consistent field approximation shows that all introduced distribution functions appear as the sources of the electromagnetic field in the Maxwell equations.

VII. EQUATION FOR THE EVOLUTION OF THE DIPOLE MOMENT DISTRIBUTION FUNCTION

Let us repeat the definition of the distribution function of dipole moment

$$d^a(\mathbf{r},\mathbf{p},t)=\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p}d\xi d\boldsymbol{\eta}\sum_{i=1}^N\xi^a\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}. \tag{51}$$

If we want to consider the evolution of the distribution function of the dipole moment, we can consider the time derivative of function $\langle\xi^a\rangle$ (51). We can consider alternative function $\langle r^a(t)\rangle = r^a f(\mathbf{r},\mathbf{p},t)+\langle\xi^a\rangle$.

The time derivative of function (51) has the following structure:

$$\partial_t d^a+\partial_{\mathbf{r}}^b\langle\xi^a v_{i,b}(t)\rangle+\partial_{\mathbf{p}}^b\langle\xi^a\dot{p}_{i,b}(t)\rangle=0. \tag{52}$$

The second term in this kinetic equation can be represented via functions (32) or (33). However, the last term in this equation requires a longer discussion similar to the analysis of interaction in the Vlasov equation presented above.

Substituting the time derivative of the momentum $\dot{p}_i^b$ into Eq. (52) [equation of motion of each particle is given by Eqs. (5)–(8)], we obtain

$$\begin{aligned}&\partial_t d^a+\partial_{\mathbf{r}}^b J_D^{ba}+q_s\left[\langle\xi^a E_{ext}^b(\mathbf{r}+\boldsymbol{\xi},t)\partial_{b,\mathbf{p}}\rangle+\frac{1}{c}\varepsilon^{bcd}\langle\xi^a v_i^c(t)B_{ext}^d(\mathbf{r}+\boldsymbol{\xi},t)\partial_{b,\mathbf{p}}\rangle\right]\\&-q_s q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'\langle\langle\xi^a(\partial_{\mathbf{r}}^b G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}'))\partial_{b,\mathbf{p}}\rangle\rangle-\frac{q_s q_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\langle\langle\xi^a v_j^b(t')(\partial_t G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}'))\partial_{b,\mathbf{p}}\rangle\rangle\\&+\frac{q_s q_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'\langle\langle\xi^a v_i^c(t)v_j^g(t')(\partial_{\mathbf{r}}^f G(t,t',\mathbf{r}+\boldsymbol{\xi},\mathbf{r}'+\boldsymbol{\xi}'))\partial_{b,\mathbf{p}}\rangle\rangle=0.\end{aligned} \tag{53}$$

Next, we make the expansion on $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$ in order to get the multipole expansion. We keep terms up to the electric quadrupole moment and find

$$\begin{aligned}&\partial_t d^a+\partial_{\mathbf{r}}^b J_D^{ba}+q_s\left[E_{ext}^b(\mathbf{r},t)\partial_{b,\mathbf{p}}\langle\xi^a\rangle+(\partial_{\mathbf{r}}^c E_{ext}^b(\mathbf{r},t))\partial_{b,\mathbf{p}}\langle\xi^a\xi^c\rangle+\frac{1}{c}\varepsilon^{bcd}B_{ext}^d(\mathbf{r},t)\partial_{b,\mathbf{p}}\langle\xi^a v_i^c(t)\rangle+\frac{1}{c}\varepsilon^{bcd}(\partial_{\mathbf{r}}^f B_{ext}^d(\mathbf{r},t))\partial_{b,\mathbf{p}}\langle\xi^a\xi^f v_i^c(t)\rangle\right]\\&-q_s q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^b G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\rangle\rangle-q_s q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^b\partial_{\mathbf{r}}^c G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi^c\rangle\rangle\\&+q_s q_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^b\partial_{\mathbf{r}}^c G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi'^c\rangle\rangle-\frac{q_s q_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_t G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a v_j^b(t')\rangle\rangle\\&-\frac{q_s q_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_t\partial_{\mathbf{r}}^c G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi^c v_j^b(t')\rangle\rangle+\frac{q_s q_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_t\partial_{\mathbf{r}}^c G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi'^c v_j^b(t')\rangle\rangle\\&+\frac{q_s q_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^f G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a v_i^c(t)v_j^g(t')\rangle\rangle+\frac{q_s q_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^f\partial_{\mathbf{r}}^l G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi^l v_i^c(t)v_j^g(t')\rangle\rangle\\&-\frac{q_s q_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'(\partial_{\mathbf{r}}^f\partial_{\mathbf{r}}^l G(t,t',\mathbf{r},\mathbf{r}'))\partial_{b,\mathbf{p}}\langle\langle\xi^a\xi'^l v_i^c(t)v_j^g(t')\rangle\rangle=0.\end{aligned} \tag{54}$$

The two-coordinate distribution functions are presented as the double brackets of the corresponding values with no usage of specific notations since part of them are introduced above, while part of them get no specific notation. Here, we make the transition to the self-consistent field approximation. To this end, we split the two-coordinate distribution functions on the product of one-coordinate distribution functions (we do it in the same way as it is done above for the equation of evolution of the scalar distribution function)

$$\begin{aligned}&\partial_t d^a+\partial_{\mathbf{r}}^b J_D^{ba}+q_s(\partial_{b,\mathbf{p}}d^a)\Bigg[E_{ext}^b(\mathbf{r},t)-q_s\left(\partial_{\mathbf{r}}^b\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t')+\frac{1}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_t G(t,t',\mathbf{r},\mathbf{r}')\langle v_j^b(t')\rangle(\mathbf{r}',\mathbf{p}')\right)\\&+q_s\partial_{\mathbf{r}}^c\left(\partial_{\mathbf{r}}^b\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')d^c(\mathbf{r}',\mathbf{p}',t')+\frac{1}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_t G(t,t',\mathbf{r},\mathbf{r}')J_D^{bc}(\mathbf{r}',\mathbf{p}',t')\right)\Bigg]\\&+q_s(\partial_{b,\mathbf{p}}q^{ac})\left[\partial_{c,\mathbf{r}}E_{ext}^b(\mathbf{r},t)-q_s\partial_{c,\mathbf{r}}\left(\partial_{\mathbf{r}}^b\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t')+\frac{1}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_t G(t,t',\mathbf{r},\mathbf{r}')\langle v_j^b(t')\rangle(\mathbf{r}',\mathbf{p}')\right)\right]\end{aligned}$$

$$+\frac{q_s}{c}\varepsilon^{bcd}(\partial_{b,\mathbf{p}}J_D^{ca})\Big[B_{ext}^d+\frac{q_{s'}}{c}\varepsilon^{dfg}\partial_{f,\mathbf{r}}\Big(\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')\langle v_j^g(t')\rangle(\mathbf{r}',\mathbf{p}')-\partial_{l,\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')J_D^{gl}(\mathbf{r}',\mathbf{p}',t')\Big)\Big]$$
$$+\frac{q_s}{c}\varepsilon^{bcd}(\partial_{b,\mathbf{p}}J_Q^{caf})\Big[\partial_{f,\mathbf{r}}B_{ext}^d+\frac{q_{s'}}{c}\varepsilon^{dfg}\partial_{f,\mathbf{r}}\partial_{l,\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')\langle v_j^g(t')\rangle(\mathbf{r}',\mathbf{p}')\Big]=0. \quad (55)$$

On this stage, we can introduce the averaged scalar and vector potentials of the electromagnetic field and corresponding representation of the integral terms within $\mathbf{E}_{int}$ and $\mathbf{B}_{int}$. This electromagnetic field obeys the Maxwell equations (48)–(50).

Finally, Eq. (55) is represented in terms of $\mathbf{E}_{int}$ and $\mathbf{B}_{int}$ as follows:

$$\partial_t d^a+\partial_{\mathbf{r}}^b J_D^{ba}+q_s(\partial_{\mathbf{p}}^b d^a)E^b+q_s(\partial_{\mathbf{p}}^b q^{ac})\partial_{\mathbf{r}}^c E^b+\frac{q_s}{c}\varepsilon^{bcd}(\partial_{\mathbf{p}}^b J_D^{ca})B^d+\frac{q_s}{c}\varepsilon^{bcd}(\partial_{\mathbf{p}}^b J_Q^{caf})\partial_{\mathbf{r}}^f B^d=0. \quad (56)$$

VIII. EQUATION FOR THE DISTRIBUTION FUNCTION OF VELOCITY

Let us repeat the definition of the considering vector distribution function (11)

$$\mathbf{j}(\mathbf{r},\mathbf{p},t)\equiv\langle\mathbf{v}_i(t)\rangle\equiv\frac{1}{\Delta}\frac{1}{\Delta_p}\int_{\Delta,\Delta_p}d\xi d\boldsymbol{\eta}\sum_{i=1}^{N/2}\mathbf{v}_i(t)\delta_{\mathbf{r}i}\delta_{\mathbf{p}i}. \quad (57)$$

If we consider the evolution of function (57), we would have four terms in the initial form of the kinetic equation

$$\partial_t\mathbf{j}(\mathbf{r},\mathbf{p},t)=\partial_t\langle\mathbf{v}_i(t)\rangle=\langle\dot{\mathbf{v}}_i(t)\rangle-\partial_r^b\langle\mathbf{v}_i(t)v_i^b(t)\rangle-\partial_p^b\langle\mathbf{v}_i(t)\dot{p}_i^b(t)\rangle. \quad (58)$$

However, we can represent the velocity of each particle $\mathbf{v}_i(t)$ via parameters $\mathbf{p}$ and $\boldsymbol{\eta}$, which do not depend on time [see Eq. (12)]

$$\partial_t\mathbf{j}(\mathbf{r},\mathbf{p},t)=\partial_t\langle\mathbf{v}+\Delta\mathbf{v}\rangle=-\partial_r^b\langle\mathbf{v}_i(t)v_i^b(t)\rangle-\partial_p^b\langle\mathbf{v}_i(t)\dot{p}_i^b(t)\rangle. \quad (59)$$

Equation (58) can be simplified to Eq. (58) using the kinetic equation for the scalar distribution function $f(\mathbf{r},\mathbf{p},t)$.

We consider Eq. (59) in more detail

$$\partial_t j^a(\mathbf{r},\mathbf{p},t)+\partial_r^b\langle v_i^a(t)v_i^b(t)\rangle+q_s\partial_p^b\langle v_i^a E_i^b(\mathbf{r}+\xi)+\varepsilon^{bcd}v_i^a v_i^c B_i^d(\mathbf{r}+\xi)\rangle-q_sq_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'\langle\langle v_i^a(t)\partial_r^b G(t,t',\mathbf{r}+\xi,\mathbf{r}'+\xi')\partial_p^b\rangle\rangle$$
$$-\frac{q_sq_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\langle\langle v_i^a(t)v_j^b(t')\partial_t G(t,t',\mathbf{r}+\xi,\mathbf{r}'+\xi')\partial_p^b\rangle\rangle+\frac{q_sq_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'$$
$$\times\langle\langle v_i^a(t)v_i^c(t)v_j^g(t')\partial_r^f G(t,t',\mathbf{r}+\xi,\mathbf{r}'+\xi')\partial_p^b\rangle\rangle=0. \quad (60)$$

Next, we represent Eq. (60) via the multipole expansion

$$\partial_t j^a(\mathbf{r},\mathbf{p},t)+\partial_r^b\langle v_i^a(t)v_i^b(t)\rangle+q_s\Big(E^b(\mathbf{r},t)\partial_p^b\langle v_i^a\rangle+(\partial_r^c E^b)\partial_p^b\langle v_i^a\xi^c\rangle+\varepsilon^{bcd}B^d\partial_p^b\langle v_i^a v_i^c\rangle+\varepsilon^{bcd}(\partial^f B^d)\partial_p^b\langle v_i^a v_i^c\xi^f\rangle\Big)$$
$$-q_sq_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_r^b G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle v_i^a(t)\rangle\rangle-q_sq_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_r^b\partial_r^c G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle\xi^c v_i^a(t)\rangle\rangle$$
$$+q_sq_{s'}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_r^b\partial_r^c G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle\xi'^c v_i^a(t)\rangle\rangle-\frac{q_sq_{s'}}{c^2}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_t G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle v_i^a(t)v_j^b(t')\rangle\rangle$$
$$+\frac{q_sq_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'\partial_r^f G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle v_i^a(t)v_i^c(t)v_j^g(t')\rangle\rangle+\frac{q_sq_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}\int d\mathbf{r}'d\mathbf{p}'\int dt'$$
$$\times\partial_r^f\partial_r^h G(t,t',\mathbf{r},\mathbf{r}')\partial_p^b\langle\langle v_i^a(t)v_i^c(t)v_j^g(t')\xi^h\rangle\rangle=0. \quad (61)$$

Finally, we present Eq. (61) in the self-consistent field approximation

$$\partial_t j^a(\mathbf{r},\mathbf{p},t)+\partial_r^b\langle v_i^a(t)v_i^b(t)\rangle+q_s\Big(E_{ext}^b(\mathbf{r},t)\partial_p^b\langle v_i^a\rangle+(\partial_r^c E_{ext}^b)\partial_p^b\langle v_i^a\xi^c\rangle+\varepsilon^{bcd}(B_{ext}^d\partial_p^b\langle v_i^a v_i^c\rangle+(\partial^f B^d)\partial_p^b\langle v_i^a v_i^c\xi^f\rangle)\Big)$$
$$-q_sq_{s'}(\partial_p^b j^a)\partial_r^b\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t')-q_sq_{s'}(\partial_p^b J_D^{ac})\partial_r^b\partial_r^c\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')f(\mathbf{r}',\mathbf{p}',t')$$
$$+q_sq_{s'}(\partial_p^b j^a)\partial_r^b\partial_r^c\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')d^c(\mathbf{r}',\mathbf{p}',t')-\frac{q_sq_{s'}}{c^2}(\partial_p^b j^a)\partial_t\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')j^c(\mathbf{r}',\mathbf{p}',t')$$
$$+\frac{q_sq_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}(\partial_p^b\langle v_i^a(t)v_i^c(t)\rangle)\partial_r^f\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')j^g(\mathbf{r}',\mathbf{p}',t')+\frac{q_sq_{s'}}{c^2}\varepsilon^{bcd}\varepsilon^{dfg}(\partial_p^b\langle v_i^a(t)v_i^c(t)\xi^h\rangle)$$
$$\times\partial_r^f\partial_r^h\int d\mathbf{r}'d\mathbf{p}'\int dt'G(t,t',\mathbf{r},\mathbf{r}')j^g(\mathbf{r}',\mathbf{p}',t')=0, \quad (62)$$

where we use several two-coordinate functions, which are presented via their correlationless limit in Appendix B [see Eqs. (B15)–(B20)].

Finally, we introduce electric $\mathbf{E}_{int}$ and magnetic $\mathbf{B}_{int}$ fields caused by the charges of the system and present the kinetic equation for the velocity distribution function in compact form:

$$\partial_t j^a + \partial^b_{\mathbf{r}}\langle v^a_i(t)v^b_i(t)\rangle + q_s E^b\partial^b_{\mathbf{r}} j^a + q_s(\partial^c_{\mathbf{r}}E^b)\partial^b_{\mathbf{p}}J^{ac}_D + \frac{q_s}{c}\varepsilon^{bcd}\Big[B^d\partial^b_{\mathbf{p}}\langle v^a_i(t)v^c_i(t)\rangle + (\partial^f B^d)\partial^b_{\mathbf{p}}\langle v^a_i(t)v^c_i(t)\xi^f\rangle\Big] = 0. \quad (63)$$

IX. ON THE EXTENDED CLOSED SET OF KINETIC EQUATIONS CONTAINING SCALAR AND VECTOR DISTRIBUTION FUNCTIONS

Evolution equations for $f(\mathbf{r},\mathbf{p},t)$, $d^a(\mathbf{r},\mathbf{p},t)$ and $j^a(\mathbf{r},\mathbf{p},t)$ [see Eqs. (47), (56), and (63)] give an incomplete set of equations. This set requires additional assumptions to make final truncation.

Let us repeat Eqs. (47), (56), and (63) together to make an additional analysis of these equations

$$\partial_t f + \mathbf{v}\cdot\nabla f + \nabla\cdot\widetilde{\mathbf{F}} + q_s\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{p}} + q_s\left(\partial^b\mathbf{E} + \frac{1}{c}\mathbf{v}\times\partial^b\mathbf{B}\right)\cdot\frac{\partial d^b_s}{\partial\mathbf{p}} + q_s\left(\partial^b\partial^c\mathbf{E} + \frac{q_s}{c}\mathbf{v}\times\partial^b\partial^c\mathbf{B}\right)\cdot\frac{\partial Q^{bc}}{\partial\mathbf{p}} + \frac{q_s}{c}\varepsilon^{abc}\left(B^c\cdot\partial_{a,\mathbf{p}}\widetilde{F}^b + \partial^d B^c\cdot\partial_{a,\mathbf{p}}J^{bd}_D + \partial^d\partial^f B^c\cdot\partial_{a,\mathbf{p}}J^{bdf}_Q\right) = 0, \quad (64)$$

and

$$\partial_t d^a + \partial^b_{\mathbf{r}}J^{ba}_D + q_s(\partial^b_{\mathbf{p}}d^a)E^b + q_s(\partial^b_{\mathbf{p}}Q^{ac})\partial^c_{\mathbf{r}}E^b + \frac{q_s}{c}\varepsilon^{bcd}(\partial^b_{\mathbf{p}}J^{ca}_D)B^d + \frac{q_s}{c}\varepsilon^{bcd}(\partial^b_{\mathbf{p}}J^{caf}_Q)\partial^f_{\mathbf{r}}B^d = 0, \quad (65)$$

and

$$\partial_t j^a + \partial^b_{\mathbf{r}}\langle v^a_i(t)v^b_i(t)\rangle + q_s E^b\partial^b_{\mathbf{r}} j^a + q_s(\partial^c_{\mathbf{r}}E^b)\partial^b_{\mathbf{p}}J^{ac}_D + \frac{q_s}{c}\varepsilon^{bcd}\Big[B^d\partial^b_{\mathbf{p}}\langle v^a_i(t)v^c_i(t)\rangle + (\partial^f B^d)\partial^b_{\mathbf{p}}\langle v^a_i(t)v^c_i(t)\xi^f\rangle\Big] = 0. \quad (66)$$

Here, we have equations for three distribution functions f, d^a, and $j^a = v^a f + \widetilde{F}^a$, but we have a number of other distribution functions like Q^{bc}, J^{bc}_D, J^{bcd}_Q, $\langle v^a_i(t)v^c_i(t)\rangle$, and $\langle v^a_i(t)v^c_i(t)\xi^f\rangle$. We need to obtain the closed set of equations at this step. Before we make this truncation, we need to provide a discussion of the physical picture hidden in these functions.

A. On covariant form of equations of motion

If we consider the dynamics of a single particle moving with velocity up to the speed of light, we can write the non-covariant form of the equation of motion $d\mathbf{p}/dt = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}/c$, where $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$ is the relativistic momentum. On the other hand, it can be represented in the covariant form $m\ddot{\xi}^\mu = (q/c)F^{\mu\nu}\dot{\xi}_\nu$, where $\xi^\mu = (ct, r)$ is the four-dimensional coordinate, $F^{\mu\nu}$ is the tensor of electromagnetic field, and the dot above the symbol denotes the derivative on the proper time s. Particularly, we have $\dot{t} = dt/ds = 1/\sqrt{1 - v^2/c^2} = \epsilon/mc^2$, where ϵ is the energy of the particle. Let us mention the structure of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ with $A^\mu = (\varphi, A)$ is the four-vector of potential of the electromagnetic field, which is composed of the scalar potential φ and vector potential $\mathbf{A}$. The covariant (under the Lorentz transformations) form of equations is preferable form of the relativistic equations. However, it is not always used. We mention it in order to specify that we consider non-covariant forms of relativistic kinetic equations. We believe that it is possible to make further generalization of presented formalism. So, the construction of the Δ-vicinity in the four-dimensional space–time will provide the required result. Here, we restrict the relativistic effects like dependence of the momentum of the particle on the velocity $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$ and the consideration of the full electromagnetic potentials including the retarding of the electromagnetic interactions.

B. Kinetic theory of fluctuations

Considering transition from the kinetic theory (the field form of the classical mechanics in the six-dimensional space) to the hydrodynamic theory (the field form of the classical mechanics in the three-dimensional space), we get the number density of particles $n(\mathbf{r},t) = \int f(\mathbf{r},\mathbf{p},t)d\mathbf{p}$ and the current of particles $\mathbf{j}(\mathbf{r},t) = n(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) = \int \mathbf{v}f(\mathbf{r},\mathbf{p},t)d\mathbf{p}$. The difference $\mathbf{v} - \mathbf{v}(\mathbf{r},t)$ is the measure of chaotic motion, which can be associated with the thermal motion. This interpretation gets closer to the thermodynamic temperature if the system gets through the process of thermalization. For the quantum systems, we also need to include other than the thermal mechanisms for the symmetric distribution of particles over the quantum states in the momentum space. Major contribution is given by the Pauli blocking existing in systems of fermions.

The average velocity $\int \mathbf{v}f(\mathbf{r},\mathbf{p},t)d\mathbf{p}$ gives the velocity field. The deviation from the average velocity $\mathbf{v} - \mathbf{v}(\mathbf{r},t)$ leads to other hydrodynamic functions, like the temperature scalar field, the pressure tensor field, etc. These functions are also smooth functions which are defined on a certain scale. It leaves the question: how fluctuations can appear in our model? Since the fluctuations is the natural phenomenon following from the individual motion of particles governed by the microscopic equations.

Basically, the chaotic motion of particles gives small variation of macroscopic functions due to the continuous unequal exchange of particles between nearest vicinities. Hence, the number density $n(\mathbf{r},t) = \int f(\mathbf{r},\mathbf{p},t)d\mathbf{p}$ (2), the distribution function $f(\mathbf{r},\mathbf{p},t)$ (4), etc., show small variation over time and coordinate. These fluctuations can be considered via the correlations like the space correlations $\overline{n(\mathbf{r},t)n(\mathbf{r}',t)}$ or the time correlations $n(\mathbf{r},t)\widetilde{n(\mathbf{r},t')}$, where we introduce an additional average on the macroscopic space scale $\overline{A} = (1/\Delta V)\int_{\Delta V}AdV$ or the average over the interval of time ΔT: $\widetilde{A} = (1/\Delta T)\int_{\Delta T}Adt$.

The distribution function is constructed on a certain scale in the coordinate and momentum space. If we want to trace the deviation from the average (particularly in the momentum space), we need to focus our attention on the deviation of momentum of all particles in Δ-vicinity from the middle point of the vicinity $\boldsymbol{\eta}$. Parameter $\Delta\mathbf{v}_i(t)$ is also associated with $\boldsymbol{\eta}$. If we imagine a Δ-vicinity with five particles (it is an imaginary example, for the illustration, see Fig. 2, real vicinity contains a large number of particles), we can count the average momentum $\overline{\mathbf{p}} = (1/5)\sum_{i=1}^{5}\mathbf{p}_i(t) \neq \mathbf{p}$ and we get a deviation from $\mathbf{p}$ (the moment of the center of the vicinity). However, the distribution

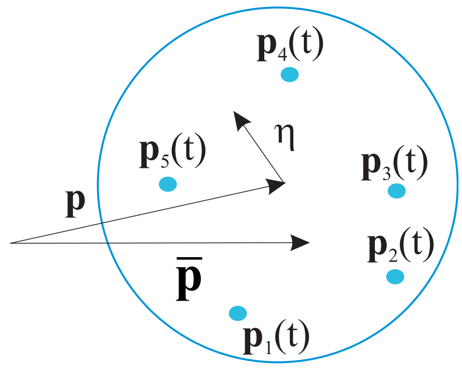

FIG. 2. The illustration of the $\Delta_{\mathbf{p}}$-vicinity in the momentum space with few numbered particles being in the vicinity. Here, $\mathbf{p}$ is the center of the vicinity, $\boldsymbol{\eta}$ is the vector scanning the vicinity, $\mathbf{p}_i(t)$ are the momentums of particles, $\overline{\mathbf{p}}$ is the average value of the momentum of particles being in the vicinity, while vector $\overline{\mathbf{p}}$ differs from the center of the vicinity $\mathbf{p}$.

function $f(\mathbf{r},\mathbf{p},t)$ counts all these particles as the particles with the momentum $\mathbf{p}$. In other words, $\sum_{i=1}^{5}\boldsymbol{\eta}_i \neq 0$ (in general, $\boldsymbol{\eta}$ is a parameter scanning the vicinity, but here we introduce a particular $\boldsymbol{\eta}_i$ as $\boldsymbol{\eta}_i(t) = \mathbf{p}_i(t) - \mathbf{p}$ the deviation of momentum of the particle being in the vicinity from the momentum of the center of vicinity). This example shows that the physical nature of fluctuations can be presented via functions $\langle \eta^a\rangle$ and $\langle \eta^a\eta^b\rangle$ (or $\langle \Delta v_i^a(t)\rangle$ and $\langle \Delta v_i^a(t)\Delta v_i^b(t)\rangle$). It explains the necessity of the account of these functions in our model. We consider the evolution equation for function $\langle \Delta v_i^a(t)\rangle$, while function $\langle \Delta v_i^a(t)\Delta v_i^b(t)\rangle$ is assumed to be approximately expressed via functions $f(\mathbf{r},\mathbf{p},t)$ and $\langle \Delta v_i^a(t)\rangle$. Here, we introduce an equation of state in order to make a truncation of the set of kinetic equations (in addition to the self-consistent field approximation).

In the chosen moment of time t, we have $\langle \Delta\mathbf{v}_i\rangle \neq 0$. However, the fluctuations happen on some timescale τ, which can be associated with the time of propagation of the average particle via the Δ-vicinity: $\tau = \sqrt[3]{\Delta}/v_0$, where the average velocity of the Δ_p-vicinity is $v_0 = pc/\sqrt{p^2+m_s^2c^2}$. We can go further and make the averaging of the kinetic equation over this time interval $\widetilde{a} = (1/\tau)\int_0^\tau a dt$. Here, we find $\langle \widetilde{\Delta\mathbf{v}_i}\rangle = 0$, but $\langle \widetilde{\Delta v_i^a\Delta v_i^b}\rangle \neq 0$. As an estimation, we can choose $\Delta\mathbf{v}_{i,max} = \sqrt[3]{\Delta_v} = \sqrt[3]{\Delta_p}c/\sqrt{\sqrt[3]{\Delta_p}^2 + m_s^2c^2}$.

C. Truncation method

Let us consider the simplified form of the kinetic equation for the vector distribution function of velocity (62). Function $j^a(\mathbf{r},\mathbf{p},t)$ can be represented via two other functions introduced above $j^a(\mathbf{r},\mathbf{p},t) = v^a\cdot f(\mathbf{r},\mathbf{p},t) + \widetilde{F}^a(\mathbf{r},\mathbf{p},t)$. Equation (62) includes function $\langle v_i^a(t)v_i^b(t)\rangle = \langle (v^a+\Delta v_i^a)(v^b+\Delta v_i^b)\rangle = v^av^b\langle 1\rangle + v^a\langle\Delta v_i^b\rangle + v^b\ \langle\Delta v_i^a\rangle + \langle\Delta v_i^a\Delta v_i^b\rangle = v^av^bf(\mathbf{r},\mathbf{p},t) + v^a\widetilde{F}^b(\mathbf{r},\mathbf{p},t) + v^b\widetilde{F}^a(\mathbf{r},\mathbf{p},t)\ + \langle\Delta v_i^a\Delta v_i^b\rangle$. Here, we extract the velocity from functions j_s^a, $j_{D,s}^{ab}$, $\langle v_i^a(t)v_i^b(t)\rangle$, and $j_{Q,s}^{abc}$ to show the Lorentz force in a more familiar way. It helps us to make the truncation as well. Finally, we represent $\langle v_i^a(t)v_i^b(t)\xi^c\rangle = v^av^b\langle\xi^c\rangle + v^a\langle\Delta v_i^b\xi^c\rangle + v^b\langle\Delta v_i^a\xi^c\rangle + \langle\Delta v_i^a\Delta v_i^b\xi^c\rangle$.

Our model includes the multipole moments in the coordinate and momentum spaces, like d^a and $\widetilde{F}^a$. The multipole moments are defined on the scale of the Δ-vicinity. Therefore, they appear to be small. On the macroscopic scale this value tends to zero. Hence, we can introduce the small parameter ε to indicate the relative value of these functions. We find $d^a \sim \varepsilon f$, $\widetilde{F}^a \sim \varepsilon f$, $Q^{ab} \sim \varepsilon^2 f$, $j^{ab} \sim \varepsilon^2 f$, etc. To make the truncation, we need to estimate the high-rank tensor distribution functions included in the model $Q^{ab} = \langle\xi^a\xi^b\rangle = \delta^{ab}\sqrt[3]{\Delta^2}f(\mathbf{r},\mathbf{p},t)\langle\Delta v^a\Delta v^b\rangle = \delta^{ab}\sqrt[3]{\Delta_v^2}f(\mathbf{r},\mathbf{p},t)$, and $j_D^{ab} = \delta^{ab}\sqrt[3]{\Delta\Delta_v}f(\mathbf{r},\mathbf{p},t)$. The third rank tensors are neglected. Function $\langle\Delta v^a\Delta v^b\rangle$ is symmetric, so it is reasonable to be proportional to δ^{ab}. However, function j_D^{ab} is not symmetric, but we assume that the deviations of the same projections of the coordinate and the momentum can be correlated. This correlation can be included in the model via nonzero value of j_D^{ab}, which can be traced in calculations of any specific problem. Further comparison with experiment can show the validity of our assumption. If no correlation is found, this function can be considered equal to zero in the future modification of the model.

We start the discussion of Eqs. (64)–(66) with the first of them (64). We can extract terms proportional to the zero degree of the small parameter ε,

$$\partial_t f + \mathbf{v}\cdot\nabla f + q_s\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial\mathbf{p}}, \tag{67}$$

which correspond to the well-known Vlasov equation. Next, we present terms proportional to the first degree of the small parameter ε,

$$q_s\left(\partial^b\mathbf{E} + \frac{1}{c}\mathbf{v}\times\partial^b\mathbf{B}\right)\cdot\frac{\partial d_s^b}{\partial\mathbf{p}} + \nabla\cdot\widetilde{\mathbf{F}} + \frac{q_s}{c}\varepsilon^{abc}B^c\cdot\partial_{a,\mathbf{p}}\widetilde{F}^b. \tag{68}$$

Other terms in Eq. (64) correspond to the second and third degree of the small parameter ε. They are neglected since we include the first correction to the Vlasov equation.

Next we consider Eq. (65). The lowest order on parameter ε is equal to one in this equation. We consider the lowest order and one correction to it. Let us show terms existing in the lowest order

$$\partial_t d^a + (\mathbf{v}\cdot\nabla)d^a + q_s(\partial_{\mathbf{p}}^b d^a)E^b + \frac{q_s}{c}\varepsilon^{bcd}v^c(\partial_{\mathbf{p}}^b d^a)B^d, \tag{69}$$

and in the next order

$$\partial_{\mathbf{r}}^b j_D^{ba} + \frac{q_s}{c}\varepsilon^{bcd}(\partial_{\mathbf{p}}^b j_D^{ca})B^d + q_s(\partial_{\mathbf{p}}^b Q^{ac})\partial_{\mathbf{r}}^c E^b + \frac{q_s}{c}\varepsilon^{bcd}v^c(\partial_{\mathbf{p}}^b Q^{af})\partial_{\mathbf{r}}^f B^d. \tag{70}$$

Let us consider Eq. (66). Function j^a splits on two terms $v^af + \widetilde{F}^a$. Obviously, Eq. (66) contains the contribution of terms (67) and (68) multiplied by v^a. Hence, all these terms are equal to zero in accordance with the evolution equation of the scalar distribution function. After this simplification, we find that the lowest order of terms in Eq. (66) on the parameter ε is equal to one. It contains the following terms:

$$\partial_t\widetilde{F}^a + (\mathbf{v}\cdot\nabla)\widetilde{F}^a + q_sE^b\partial_{\mathbf{p}}^b\widetilde{F}^a + \frac{q_s}{c}\varepsilon^{bcd}v^cB^d\partial_{\mathbf{p}}^b\widetilde{F}^a. \tag{71}$$

In the next order on ε, we obtain

$$\partial_{\mathbf{r}}^b\langle\Delta v^a\Delta v^b\rangle + q_s(\partial_{\mathbf{r}}^c E^b)\partial_p^b j_D^{ac} + \frac{q_s}{c}\varepsilon^{bcd}B^d\partial_{\mathbf{p}}^b\langle\Delta v^a\Delta v^c\rangle + \frac{q_s}{c}\varepsilon^{bcd}v^c(\partial_{\mathbf{r}}^f B^d)\partial_{\mathbf{p}}^b j_D^{af}. \tag{72}$$

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In this section, we introduce three kinds of correlations. First, it is the coordinate-coordinate correlation $Q^{ab} = \langle \xi^a \xi^b \rangle = \delta^{ab}\sqrt[3]{\Delta^2} f(\mathbf{r}, \mathbf{p}, t)$. Second, it is the momentum-momentum correlation $\langle \Delta v^a \Delta v^b \rangle = \delta^{ab}\sqrt[3]{\Delta_v^2} f(\mathbf{r}, \mathbf{p}, t)$. Finally, it is the coordinate-momentum correlation $j_D^{ab} = \delta^{ab}\sqrt[3]{\Delta\Delta_v} f(\mathbf{r}, \mathbf{p}, t)$. Below, we present our results for the dynamic of the small amplitude perturbations, which shows that the coordinate-momentum correlation leads to an nonphysical instability (at the analysis of the dispersion dependence of the Langmuir waves). Therefore, we assume below that there is no coordinate-momentum correlation $j_D^{ab} = 0$. This nonphysical part is not demonstrated below.

D. Final set of kinetic equations

Let us combine together the final parts of Eqs. (64)–(66) with the equations of state

$$\partial_t f + \mathbf{v}\cdot\nabla f + q_s\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{p}} + q_s\left(\partial^b \mathbf{E} + \frac{1}{c}\mathbf{v}\times\partial^b\mathbf{B}\right)\cdot\frac{\partial d_s^b}{\partial \mathbf{p}} + \nabla\cdot\widetilde{\mathbf{F}} + \frac{q_s}{c}\varepsilon^{abc}B^c\cdot\partial_{a,\mathbf{p}}\widetilde{F}^b = 0, \tag{73}$$

$$\partial_t d^a + (\mathbf{v}\cdot\nabla)d^a + q_s(\partial^b_{\mathbf{p}} d^a)E^b + \frac{q_s}{c}\varepsilon^{bcd}v^c(\partial^b_{\mathbf{p}} d^a)B^d + q_s\sqrt[3]{\Delta^2}(\partial^b_{\mathbf{p}} f)\partial^a_{\mathbf{r}} E^b + \frac{q_s}{c}\sqrt[3]{\Delta^2}\varepsilon^{bcd}v^c(\partial^b_{\mathbf{p}} f)\partial^a_{\mathbf{r}} B^d = 0, \tag{74}$$

and

$$\partial_t \widetilde{F}^a + (\mathbf{v}\cdot\nabla)\widetilde{F}^a + q_s E^b\partial^b_{\mathbf{p}}\widetilde{F}^a + \frac{q_s}{c}\varepsilon^{bcd}v^c B^d\partial^b_{\mathbf{p}}\widetilde{F}^a + \sqrt[3]{\Delta_v^2}\partial^a_{\mathbf{r}} f - \frac{q_s}{c}\sqrt[3]{\Delta_v^2}\varepsilon^{abd}B^d\partial^b_{\mathbf{p}} f = 0. \tag{75}$$

The electromagnetic field in Eqs. (73)–(75) obeys the following Maxwell equations:

$$\nabla\cdot\mathbf{B}_{int} = 0, \quad \nabla\times\mathbf{E}_{int} = -\frac{1}{c}\partial_t\mathbf{B}_{int}, \tag{76}$$

$$(\nabla\times\mathbf{B}_{int})^a = \frac{1}{c}\partial_t E^a_{int} + \frac{4\pi}{c}\sum_s q_s\left(\int v^a f_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \partial^b\int v^a d^b_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \int \widetilde{F}^a_s(\mathbf{r},\mathbf{p},t)d\mathbf{p}\right), \tag{77}$$

$$\nabla\cdot\mathbf{E}_{int} = 4\pi\sum_s q_s\left(\int f_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \partial^b\int d^b_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} + \frac{1}{2}\sqrt[3]{\Delta^2}\partial^b\partial^b\int f(\mathbf{r},\mathbf{p},t)d\mathbf{p}\right), \tag{78}$$

which appear as corresponding modification of Eqs. (48)–(50).

Reference 45 is focused on the method of microscopic justification of the Vlasov kinetic equation and the hydrodynamic equations for the plasmas. It contains the justification of the mean-field approximation in terms of the deterministic viewpoint. So, it gives a novel background for the well-known models. The appearance of "novel" functions related to the electric dipole moment of the physically infinitesimal volume is demonstrated in Ref. 45, but their contribution is dropped. In this paper, we focus on the contribution of the novel function and on the construction of the closed model, which is a generalization of the Vlasov kinetic equation. This model is derived using methods developed in Ref. 45.

X. THE SPIN-ELECTRON-ACOUSTIC WAVES PROPAGATION IN THE SPIN POLARIZED ELECTRON GAS OF HIGH DENSITY

The degenerate macroscopically motionless (being in the equilibrium state) electron gas is described via the Vlasov kinetic equation for each spin projection of electrons[30]

$$\partial_t f_s + \mathbf{v}\cdot\nabla f_s + q_s\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_s}{\partial\mathbf{p}} = 0. \tag{79}$$

Subindex s corresponds to the electrons with the spin-up $s=\uparrow$ or spin-down $s=\downarrow$. We consider the small perturbations of the equilibrium state $f_s = f_{0s} + \delta f_s$, where f_{0s} is the equilibrium distribution function, and δf_s is the perturbation of the distribution function, which is chosen as the plane wave in the coordinate space $\delta f_s = F_s e^{-\imath\omega t + \imath k_z z}$, with the constant amplitude F_s. It leads to the linearized kinetic equation

$$-\imath(\omega - k_z v_z)\delta f_s + q_s\delta\mathbf{E}\cdot\frac{\partial f_{0s}}{\partial\mathbf{p}} = 0, \tag{80}$$

where $\delta\mathbf{B} = 0$ for the longitudinal waves.

Equilibrium distribution functions for each subspecies of electrons is chosen as the Fermi step $f_{0s} = \theta(p_{Fs} - p)/(2\pi\hbar)^3$, where $p_{Fs} = (6\pi n_{0s})^{1/3}\hbar$ is the radius of the Fermi sphere in the momentum space for species s.

We consider the longitudinal waves; hence, the perturbation of the electric field is parallel to the direction of the wave propagation $\mathbf{k}\parallel\delta\mathbf{E}$.

Equation (80) gives the expression for the perturbation of the distribution function of electrons

$$\delta f_s = -\imath q_s(\mathbf{v}\cdot\delta\mathbf{E})\frac{\partial f_{0s}}{\partial\varepsilon}\frac{1}{\omega - k_z v_z}, \tag{81}$$

where

$$\frac{\partial f_{0s}}{\partial\varepsilon} = \frac{\partial f_{0s}}{\partial p}\frac{\partial p}{\partial\varepsilon} = -\frac{1}{(2\pi\hbar)^3}\delta(p - p_{Fs})\frac{\varepsilon}{pc^2}. \tag{82}$$

Expression (81) allows us to calculate the perturbations of the number density of the degenerate electrons

$$\delta n_s = \int\delta f_s(\mathbf{r},\mathbf{p},t)d\mathbf{p} = -\frac{2\pi\imath q_s p_{Fs}^2\delta E_z}{(2\pi\hbar)^3 k_z v_{Fs}}\left[2 + \frac{\omega}{k_z v_{Fs}} ln\left(\frac{\omega - k_z v_{Fs}}{\omega + k_z v_{Fs}}\right)\right]. \tag{83}$$

It leads to the dispersion equation

$$1 + \frac{3}{2}\frac{\omega_{Le}^2}{k_z^2 c^2}\sum_{s=u,d}\frac{n_{0s}}{n_{0e}}\gamma_{Fs}\frac{m_s^2 c^2}{p_{Fs}^2}\left[2 + \frac{\omega}{k_z v_{Fs}} ln\left(\frac{\omega - k_z v_{Fs}}{\omega + k_z v_{Fs}}\right)\right] = 0. \tag{84}$$

The presented dispersion equation is found for the motionless ions, but we have two species of particles: the spin-up electrons and the spin-down electrons. Both species of electrons are degenerate, but they have different equilibrium number densities. So, we have nonzero equilibrium spin polarization. The spin polarization is caused by the external magnetic field. However, it is not included in Eq. (80), since it does not affect the final result (81) for the waves propagating parallel to the magnetic field.

Equation (84) gives two solutions: the Langmuir wave and the SEAW. The SEAW has been studied in different regimes and using different methods. It is considered within the nonrelativistic hydrodynamics,[29] the nonrelativistic kinetics,[30] and the relativistic hydrodynamics.[41] Here, we present the relativistic kinetic theory of the SEAW. The numerical analysis of Eq. (84) shows that the dispersion dependencies of the Langmuir wave and the SEAW get close to each other at the large wave vectors, which leads to an instability in the short-wavelength limit. However, it can be subject of another work specified on the instabilities in the degenerate plasmas related to the SEAWs, while this paper is focused on the derivation of the extended kinetic model.

XI. LANGMUIR WAVES AND SEAWs UNDER INFLUENCE OF THE DIPOLAR DISTRIBUTION FUNCTION EVOLUTION

In this section, we consider the same equilibrium condition like in Sec. X. However, we consider the dynamics of the vector distribution functions. We also assume $d^b_{0s}=0$ and $\widetilde{F}^a_{0s}=0$. We choose this equilibrium condition since we do not suggest any physical mechanisms to create nonzero values of these functions in the equilibrium.

Let us present the linearized kinetic equations on the small amplitude perturbations for the monochromatic plane waves:

$$-\imath(\omega-k_zv_z)\delta f_s+q_s\delta\mathbf{E}\cdot\frac{\partial f_{0s}}{\partial\mathbf{p}}+\nabla\cdot\delta\widetilde{\mathbf{F}}=0, \qquad (85)$$

$$-\imath(\omega-k_zv_z)\delta d^a+q_s\sqrt[3]{\Delta^2}(\partial^b_{\mathbf{p}}f_{0s})\partial^a_{\mathbf{r}}\delta E^b=0, \qquad (86)$$

and

$$-\imath(\omega-k_zv_z)\delta\widetilde{F}^a+\sqrt[3]{\Delta_v^2}\partial^a_{\mathbf{r}}\delta f_s=0. \qquad (87)$$

We also present the linearized form of the Poisson equation

$$\imath k_z\delta E_z=4\pi q_e\sum_s\left[\left(1-\frac{1}{2}\sqrt[3]{\Delta^2}k_z^2\right)\int\delta f_s d\mathbf{p}+\imath k_z\int\delta d^z_s d\mathbf{p}\right]. \qquad (88)$$

Equations (85)–(87) give the following expressions for the perturbations of the distribution functions:

$$\delta\widetilde{\mathbf{F}}=\frac{\sqrt[3]{\Delta_v^2}\nabla\delta f_s}{\imath(\omega-k_zv_z)}, \qquad (89)$$

$$\delta d^z_s=\frac{q_s\sqrt[3]{\Delta^2}k_z\delta E^z\frac{\partial f_{0s}}{\partial p_z}}{(\omega-k_zv_z)}, \qquad (90)$$

and

$$\delta f_s=\frac{q\delta E_z\frac{\partial f_{0s}}{\partial p_z}}{\imath(\omega-k_zv_z)}\frac{1}{1-\frac{\sqrt[3]{\Delta_v^2}k_z^2}{(\omega-k_zv_z)^2}}. \qquad (91)$$

Substituting solutions (89)–(91) in the linearized Poisson equation (88) gives the dispersion equation for the longitudinal waves as

$$-4\pi q_e^2\sum_s\left[\left(1-\frac{1}{2}\sqrt[3]{\Delta^2}k_z^2\right)\int\left(\frac{(\omega-k_zv_z)\frac{\partial f_{0s}}{\partial p_z}}{(\omega-k_zv_z)^2-k_z^2\sqrt[3]{\Delta_v^2}}\right)d\mathbf{p}+k_z^2\sqrt[3]{\Delta^2}\int\frac{\frac{\partial f_{0s}}{\partial p_z}}{(\omega-k_zv_z)}d\mathbf{p}\right]=k_z. \qquad (92)$$

In Eq. (92), we see the contribution of the vector distribution functions via coefficients $\sqrt[3]{\Delta^2}$ and $\sqrt[3]{\Delta_v^2}$. Moreover, Eqs. (89) and (90) show that the perturbations of the vector distribution functions are proportional to $\sqrt[3]{\Delta^2}$ or $\sqrt[3]{\Delta_v^2}$.

We substitute the equilibrium distribution function [see Eq. (82) and text above Eq. (81)] and integrate over the module of momentum p and angle φ,

$$\sum_s\frac{3}{2}\frac{1}{\gamma_{Fs}v_{Fs}}\omega_{Ls}^2\left[\left(1-\frac{1}{2}\sqrt[3]{\Delta^2}k_z^2\right)\int\left(\frac{(\omega-k_zv_{Fs}\cos\theta)\cos\theta\sin\theta d\theta}{(\omega-k_zv_{Fs}\cos\theta)^2-k_z^2\sqrt[3]{\Delta_v^2}}\right)-k_z^2\sqrt[3]{\Delta^2}\int\frac{\cos\theta\sin\theta d\theta}{(\omega-k_zv_{Fs}\cos\theta)}\right]=k_z, \qquad (93)$$

where $\varepsilon_{Fs}=\sqrt{p_{Fs}^2c^2+m_e^2c^4}=\gamma_{Fs}m_ec^2$.

After integration, we obtain the following form of the dispersion equation:

$$\sum_s\frac{3}{2}\frac{1}{\gamma_{Fs}v_{Fs}^2}\omega_{Ls}^2\left[\left(1-\frac{1}{2}\sqrt[3]{\Delta^2}k_z^2\right)\times\left[2+\frac{\sqrt[3]{\Delta_v}k_z}{2k_zv_{Fs}}\ln\left(\frac{\omega^2-(k_zv_{Fs}-\sqrt[3]{\Delta_v}k_z)^2}{\omega^2-(k_zv_{Fs}+\sqrt[3]{\Delta_v}k_z)^2}\right)+\frac{\omega}{2k_zv_{Fs}}\ln\left(\frac{(\omega-k_zv_{Fs})^2-\sqrt[3]{\Delta_v^2}k_z^2}{(\omega+k_zv_{Fs})^2-\sqrt[3]{\Delta_v^2}k_z^2}\right)\right]-k_z^2\sqrt[3]{\Delta^2}\left[2+\frac{\omega}{k_zv_{Fs}}\ln\left(\frac{\omega-k_zv_{Fs}}{\omega+k_zv_{Fs}}\right)\right]\right]=-k_z^2, \qquad (94)$$

which is a generalization of Eq. (84).

In order to consider the Langmuir waves and SEAWs we need to derive two different limits of Eq. (94).

A. Langmuir waves under influence of the dipolar distribution function evolution

To obtain the dispersion dependence of the Langmuir waves from Eq. (94), we need to consider the high-frequency regime $\omega\gg k_zv_{Fs}$. However, we have additional combination of parameters $\sqrt[3]{\Delta_v}k_z$, which should be compared with ω and k_zv_{Fs}. Parameter $\sqrt[3]{\Delta_v}$ is introduced as a small value in comparison with the characteristic velocity of the system v_{Fs}. Therefore, we have $\omega\gg k_zv_{Fs}\gg\sqrt[3]{\Delta_v}k_z$. It leads to

$$\ln\left(\frac{\omega^2-(k_zv_{Fs}-\sqrt[3]{\Delta_v}k_z)^2}{\omega^2-(k_zv_{Fs}+\sqrt[3]{\Delta_v}k_z)^2}\right)\approx 4\frac{k_zv_{Fs}}{\omega}\frac{\sqrt[3]{\Delta_v}k_z}{\omega}$$

and

$$\ln\left(\frac{(\omega-k_z v_{Fs})^2-\sqrt[3]{\Delta_v^2}k_z^2}{(\omega+k_z v_{Fs})^2-\sqrt[3]{\Delta_v^2}k_z^2}\right)$$
$$\approx -4\frac{k_z v_{Fs}}{\omega}\times\left(1+\frac{1}{3}\frac{k_z^2 v_{Fs}^2}{\omega^2}+\frac{\sqrt[3]{\Delta_v^2}k_z^2}{\omega^2}+\frac{1}{5}\frac{k_z^4 v_{Fs}^4}{\omega^4}\right).$$

Let us also remind the expansion existing for the logarithm in the last term

$$\ln\left(\frac{\omega-k_z v_{Fs}}{\omega+k_z v_{Fs}}\right)\approx -2\frac{k_z v_{Fs}}{\omega}\left(1+\frac{1}{3}\frac{k_z^2 v_{Fs}^2}{\omega^2}+\frac{1}{5}\frac{k_z^4 v_{Fs}^4}{\omega^4}\right).$$

We substitute these expansions in the dispersion equation (94) and find the dispersion equation for the Langmuir waves in the long-wavelength limit as

$$1=\frac{1}{\omega^2}\sum_s\left(1-\frac{3}{2}\sqrt[3]{\Delta^2}k_z^2\right)\frac{\omega_{Ls}^2}{\gamma_{Fs}}\left(1+\frac{3}{5}\frac{k_z^2 v_{Fs}^2}{\omega^2}\right). \quad (95)$$

Two groups of terms in Eq. (94) are combined together. It leads to the change of coefficient in front of $\sqrt[3]{\Delta^2}$ from 1/2 to 3/2. We also see that the contribution of $\sqrt[3]{\Delta_v^2}$ is canceled in the considered approximation. In the zeroth order on k_z, we find $\omega^2=\omega_{Ls}^2/\gamma_{Fs}$. The contribution of small corrections in the second order on k_z^2 is found as

$$\omega^2=\frac{\omega_{Le}^2}{\gamma_{Fe}}\left(1-\frac{3}{2}\sqrt[3]{\Delta^2}k_z^2\right)+\frac{3}{5}k_z^2 v_{Fe}^2. \quad (96)$$

The contribution of $\sqrt[3]{\Delta^2}$ leads to the decrease in the group velocity related to the Fermi pressure/Pauli blocking given by the last term.

The estimation of the delta-vicinity is given in Ref. 45 for two regimes. First, the hydrodynamic regime requires the number density to be a continuous function. It leads to $\sqrt[3]{\Delta}\approx\sqrt{ar_D}$, where a is the average interparticle distance $a\sim n_{0e}^{-1/3}$, and $r_D=\sqrt{T/(8\pi n_{0e}e^2)}$ is the Debye radius, for the nondegenerate systems. The temperature of the system T can be replaced with the Fermi temperature $T_{Fe}=E_{Fe}$ for the degenerate systems, where E_{Fe} is the Fermi energy (for the nonrelativistic systems, in the relativistic case we need to exclude the rest energy $T_{Fe}=E_{Fe}-m_ec^2$). In the second (kinetic) regime, we require the distribution function to be a continuous function. Basically, it means that the number density should be a continuous function for an interval of velocities. It leads to the larger volume of the delta-vicinity in the coordinate space $\sqrt[3]{\Delta_{kin}}\approx r_D$. It also corresponds to estimation given by Klimontovich.[1]

Using these estimations of the delta-vicinity, we can rewrite Eq. (96) in the dimensionless form for the kinetic regime $w^2=\frac{1}{\gamma_{Fe}}\left(1-\frac{3}{2}\frac{r_D^2\omega_{Le}^2}{v_{Fe}^2}\kappa^2\right)+\frac{3}{5}\kappa^2$, where the contribution of the delta-vicinity is maximal, $w=\omega/\omega_{Le}$ is the dimensionless frequency, and $\kappa=k_z v_{Fe}/\omega_{Le}$ is the dimensionless wave vector module.

The Debye radius has different expressions in the relativistic and nonrelativistic regimes. Let us start with the nonrelativistic regime, where $E_{Fe}=m_e v_{Fe}^2/2$ and $r_D^2\omega_{Le}^2/v_{Fe}^2=1/4$. Hence, the dispersion dependence simplifies to $w^2=1+\left(\frac{3}{5}-\frac{3}{8}\right)\kappa^2$, where the term proportional to 3/8 is caused by the delta-vicinity contribution. Let us repeat that we consider the maximal value for the delta-vicinity volume. If we make reestimation for the hydrodynamic regime, we get the additional small parameter in the last term $w^2=1+\left(\frac{3}{5}-\frac{3}{8}\frac{a}{r_D}\right)\kappa^2$.

Next, we consider the relativistic regime and choose the maximal value of the delta-vicinity volume. In this regime, we have $E_{Fe}=\gamma_{Fe}m_ec^2$ and $\frac{r_D^2\omega_{Le}^2}{v_{Fe}^2}=(1/2)(\gamma_{Fe}-1)(v_{Fe}^2/c^2)^{-1}$.

In Fig. 3, we present the numerical analysis of the dimensionless form of Eq. (96) for the kinetic regime $w^2=\frac{1}{\gamma_{Fe}}[1-\frac{3}{4}(\gamma_{Fe}-1)\times\left(\frac{v_{Fe}^2}{c^2}\right)^{-1}\kappa^2]+\frac{3}{5}\kappa^2$. Figure 4 specifies the change of the coefficient in front of κ^2. The presented results show that the maximal estimation of

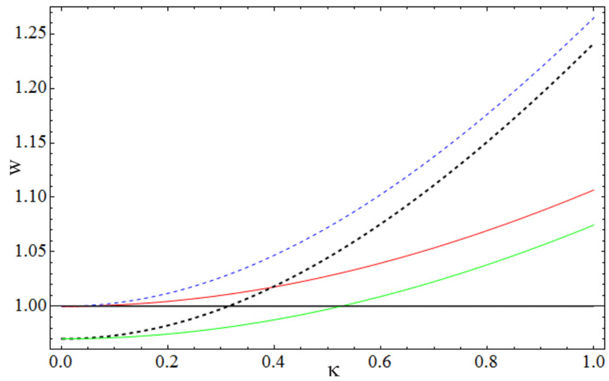

FIG. 3. The dispersion dependence of Langmuir waves is presented as the dependence of the dimensionless frequency $w=\omega/\omega_{Le}$ on the dimensionless wave vector $\kappa=k_z v_{Fe}/\omega_{Le}$. It is presented for two values of the number density. The value $n_1=10^{22}$ cm^{-3} corresponds to nonrelativistic regime. It is presented by blue (upper dashed) and red (upper continuous) lines. The value $n_2=2.7\times10^{28}$ cm^{-3} demonstrates the contribution of the relativistic effects. It is presented by black (lower dashed) and green (lower continuous) lines. The dashed lines correspond to no account of the delta-vicinity. The continuous lines show Eq. (96) containing the contribution of the delta-vicinity.

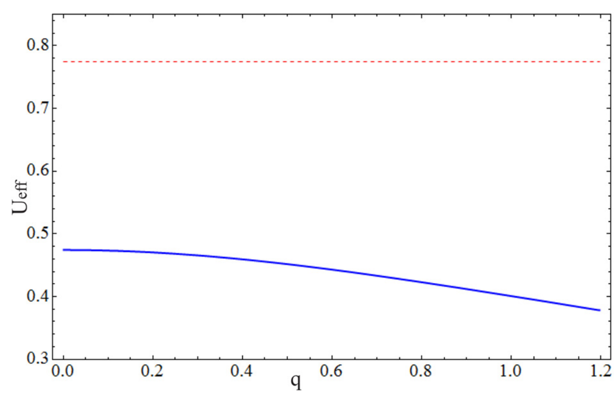

FIG. 4. The dimensionless form of the dispersion dependence of the Langmuir waves can be represented as $\xi^2=\frac{1}{\gamma_{Fe}}+U_{eff}^2\kappa^2$. Effective velocity U_{eff} as the function of the dimensionless concentration $q=p_{Fe}/m_ec\sim n_{0e}^{1/3}$ is plotted here. The upper red dashed line corresponds to the zero contribution of the delta-vicinity $U_{eff}^2=3/5$. The lower continuous line corresponds to the contribution of the delta-vicinity in accordance with Eq. (96) for the maximal estimation of value of $\sqrt[3]{\Delta_{kin}}\approx r_D$ in the kinetic regime.

the coordinate-space delta-vicinity appearing in the kinetic regime[1] gives a relatively large change in the dispersion dependence.

This large value of the vicinity volume appearing in the kinetic regime happens due to requirement of the smoothness (or the continuously differentiable) of the distribution function. If we keep the number density as the continuously differentiable function at a discontinuous distribution function, then we have the hydrodynamic regime with the smaller delta-vicinity. Therefore, it decreases the contribution of the delta-vicinity in the dispersion dependence of the Langmuir waves. Let us to give a reestimation of the volume of the delta-vicinity. First, we repeat our previous arguments. We consider $\sqrt{a/r_D}$ as a small parameter $\epsilon = \sqrt{a/r_D}$. For the characteristic macroscopic distance r_D, we have a small distance $\epsilon r_D = \sqrt{ar_D}$. It is used as the radius of the delta-vicinity in the hydrodynamics $\sqrt[3]{\Delta_{hydr}} = \epsilon r_D = \sqrt{ar_D}$. Next, we increase the scale to get the continuous distribution function in the kinetic regime $\sqrt[3]{\Delta_{kin}} = r_D$. On the other hand, we can consider a scale that has an intermediate value between a and $\sqrt{ar_D}$. It is the geometric mean of these scales $\sqrt{a\sqrt{ar_D}} = \sqrt[4]{a^3 r_D}$. It can be interpreted as another radius of the delta-vicinity in the hydrodynamics $\sqrt[3]{\tilde{\Delta}_{hydr}} = \sqrt[4]{a^3 r_D}$. The corresponding reestimation of the delta-vicinity radius for the kinetic regime gives $\sqrt[3]{\tilde{\Delta}_{kin}} = \sqrt{ar_D}$. It provides the decrease in difference in the continuous and dashed lines in Figs. 3 and 4.

B. SEAWs under influence of the dipolar distribution function evolution

Let us point out that the SEAW is the additional wave solution appearing in the electron gas in addition to the Langmuir wave. It can exist in the partially spin-polarized electron gas. However, it does not exist in the electron gas with the zero spin polarization. Relativistic SEAWs considered in terms of the Vlasov equation with no account of the delta-vicinity are studied in Ref. 41. Here, we point out some features related to the delta-vicinity and the evolution of functions $\mathbf{d}(\mathbf{r}, \mathbf{p}, t)$ and $\widetilde{\mathbf{F}}(\mathbf{r}, \mathbf{p}, t)$.

The SEAWs correspond to the intermediate frequency interval $k_z v_{Fd} \gg \omega \gg k_z v_{Fu} \gg \sqrt[3]{\Delta_v} k_z$. Therefore, we need to consider the opposite regime of expansion of ln functions in Eq. (94) for the spin-down electrons:

$$\ln\left(\frac{\omega^2 - (k_z v_{Fs} - \sqrt[3]{\Delta_v} k_z)^2}{\omega^2 - (k_z v_{Fs} + \sqrt[3]{\Delta_v} k_z)^2}\right) \approx 2\left(\frac{\omega}{k_z v_{Fs}} - \frac{\sqrt[3]{\Delta_v} k_z}{k_z v_{Fs}}\right)$$

and

$$\ln\left(\frac{(\omega - k_z v_{Fs})^2 - \sqrt[3]{\Delta_v^2} k_z^2}{(\omega + k_z v_{Fs})^2 - \sqrt[3]{\Delta_v^2} k_z^2}\right) \approx -4\frac{\omega}{k_z v_{Fs}}.$$

Let us also remind the expansion existing for the logarithm in the last term

$$\ln\left(\frac{\omega - k_z v_{Fs}}{\omega + k_z v_{Fs}}\right) \approx -2\frac{\omega}{k_z v_{Fs}}.$$

We need to use these expansions for the spin-down electrons. However, we use expansions given in Sec. XI A for the spin-up electrons. We substitute it in the dispersion equation (94) in order to obtain the simplified expression for the dispersion equation for the SEAWs,

$$k_z = \frac{3}{2}\frac{\omega_{Ld}^2}{\gamma_{Fd}}\frac{1}{v_{Fd}^2 k_z}\left(-2 - \frac{\omega}{k_z v_{Fd}}\frac{\sqrt[3]{\Delta_v}}{v_{Fd}} + \frac{\sqrt[3]{\Delta_v^2}}{v_{Fd}^2} + \frac{\omega^2}{k_z^2 v_{Fd}^2} + 3\sqrt[3]{\Delta^2} k_z^2\right) + \frac{\omega_{Lu}^2}{\gamma_{Fu}}\frac{k_z}{\omega^2}\left(1 - \frac{3}{2}\sqrt[3]{\Delta^2} k_z^2 + \frac{3}{5}\frac{k_z^2 v_{Fu}^2}{\omega^2}\right). \tag{97}$$

First, we need to consider the zeroth approximation on the small parameters

$$1 + 3\frac{\omega_{Ld}^2}{\gamma_{Fd}}\frac{1}{v_{Fd}^2 k_z^2} = \frac{\omega_{Lu}^2}{\gamma_{Fu}\omega^2}. \tag{98}$$

In the long-wavelength limit, we can also drop the first term on the left-hand side

$$\omega^2 = \frac{1}{3}\frac{\gamma_{Fd}\omega_{Lu}^2}{\gamma_{Fu}\omega_{Ld}^2} v_{Fd}^2 k_z^2. \tag{99}$$

Next, we consider corrections to solution (98). We include corrections on the wave vector appearing from Eq. (98). We also include corrections appearing from the higher order of expansions on the well-known small parameters demonstrated in Eq. (97). Finally, we include corrections related to the evolution of the vector distribution functions [they are also demonstrated in Eq. (97)]. We obtain this solution from Eq. (97) by the iteration method, meaning we place solution (99) instead of ω in the small terms in Eq. (97). Hence, we find

$$\omega^2 = \frac{1}{3}\frac{\gamma_{Fd}\omega_{Lu}^2}{\gamma_{Fu}\omega_{Ld}^2} v_{Fd}^2 k_z^2\left(1 - \frac{1}{3}\frac{\gamma_{Fd} v_{Fd}^2 k_z^2}{\omega_{Ld}^2} - 3\sqrt[3]{\Delta^2} k_z^2 + \frac{9}{5}\frac{\gamma_{Fd}\omega_{Lu}^2}{\gamma_{Fu}\omega_{Ld}^2}\frac{v_{Fu}^2}{v_{Fd}^2} - \frac{1}{6}\frac{\gamma_{Fd}\omega_{Lu}^2}{\gamma_{Fu}\omega_{Ld}^2} - \frac{1}{2\sqrt{3}}\frac{\gamma_{Fd}\omega_{Lu}}{\gamma_{Fu}\omega_{Ld}}\frac{\sqrt[3]{\Delta_v}}{v_{Fd}} - \frac{1}{2}\frac{\sqrt[3]{\Delta_v^2}}{v_{Fd}^2}\right). \tag{100}$$

Here, we see two kinds of corrections: one of them depends on the wave vector k_z, and the other does not depend on the wave vector k_z. Both kinds are composed of the well-known terms, which are related to the dynamics of the scalar distribution function, and novel terms, which are caused by the vector distribution functions.

Let us consider these corrections in more detail. Four terms (from the second term to the fifth term) on the right-hand side do not depend on the wave vector. Two of them are well-known terms (the second and third terms). They have different signs, but obviously the negative term is larger $v_{Fd}^2 \gg v_{Fu}^2$. Two novel terms are negative; hence, novel terms increase the contribution of the well-known terms. Next we consider two last terms in Eq. (100). They depend on k_z and have negative signs. Therefore, the contribution of the vector distribution functions via the coefficient $\sqrt[3]{\Delta^2}$ strengthens the decrease in the group velocity. The last conclusion happens in contrast with the result obtained for the Langmuir wave, where the term proportional to $\sqrt[3]{\Delta^2}$ decreased the group velocity and gave the competition to the Fermi pressure.

In Eq. (100), we have three terms containing the delta-vicinity. Two last terms explicitly contain the small parameter $\sqrt[3]{\Delta_v} \sim \epsilon v_{Fe}$. The third term $3\sqrt[3]{\Delta^2} k_z^2$ is more sensitive to the estimations of the coordinate part of the delta-vicinity $\sqrt[3]{\Delta}$. However, this term is proportional to the wave vector k. Equation (100) is obtained in the long-wavelength limit. Hence, we consider small wave vectors k and this term is relatively small in comparison with the first term.

XII. CONCLUSION

The detailed derivation of kinetic equations for the relativistic plasmas has been presented. It concludes in the set of three kinetic equations (for each species of particles): the Vlasov kinetic equation for the scalar distribution function, the equation of evolution of the vector distribution function of the electric dipole moment (the fluctuation of local center of mass of the particles being in the physically infinitesimal volume), and the equation of evolution of the vector distribution function of the velocity (the fluctuation of average momentum of the particles being in the physically infinitesimal volume). This closed set of equations appears as the result of truncation of the quasi-infinite set of kinetic equations (number of degrees of freedom is restricted due to the finite number of particles N), which extends via the account of two-, three-, and more coordinate distribution functions and higher multipole moments of one-coordinate (along with two-, three-, and more coordinate) distribution functions. The developed truncation method represents the one-coordinate tensor distribution functions via the one-coordinate scalar and vector distribution functions. The radius of the physically infinitesimal volume explicitly enters the coefficients appearing in this representation. This methodology based on the small value of the physically infinitesimal volume in comparison with the macroscopic scales. This truncation appears in addition to the splitting of the two-coordinate distribution function, which is usually part of the transition to the mean-field approximations. All described steps have been made within the kinetic theory in terms of the distribution functions. So, no transition to hydrodynamics or no calculations of moments of the distribution functions have been made.

The derivation method shows the representation of the microscopic motion of the individual particles in terms of the macroscopic distribution functions. It gives the representation of the deterministic classical mechanics in terms of the evolution of the collective functions. No probabilistic approaches are used for the derivation or interpretation of presented equations and found functions.

Equations have been found for the relativistic plasmas meaning both the large temperatures of the system (large velocities of the chaotic motion) and the large velocities of the ordered motion. The motion of particles with the velocities close to the speed of light requires the account of full retarding potentials for the electromagnetic field created by particles. It reflects in the full Maxwell equations obtained in the self-consistent field approximation. The Maxwell equations also include the distribution function of the electric dipole moment, distribution function of the velocity, etc., as the sources of the electromagnetic field.

The model is found for the hot plasmas, but it can be used for the degenerate electron gas with the Fermi velocity close to the speed of light. Hence, it is applied to the spin-electron acoustic waves in the extremely dense plasmas. The dispersion dependence of the Langmuir waves has been found under the influence of the vector distribution functions. Hence, the effect of scale corresponding to the transition to the macroscopic level of description reveals in the characteristics of waves. It gives an additional term containing the radius of the coordinate part of the physically infinitesimal volume. This effect gives a decrease in the group velocity of the Langmuir waves. Hence, it allows to check the chosen method of truncation together with the possible values of the physically infinitesimal volume. The influence of the vector distribution functions on the spin-electron acoustic waves has been considered as well. The spin-electron acoustic waves, in contrast to the Langmuir waves, depend on the radiuses of coordinate and momentum parts of the physically infinitesimal volume. However, the dependence on the momentum part of the physically infinitesimal volume is relatively small. Further estimations of the physically infinitesimal volume can require the consideration of the dispersion dependencies of other waves existing in the plasmas both in the relativistic and nonrelativistic regimes.

Here, the waves have been considered as the perturbations of the equilibrium state with the zero values of the vector distribution functions. It corresponds to the realistic equilibrium conditions. However, nowadays, there are studies of plasmas that have not reached the equilibrium state. They are usually described by the non-Maxwellian electron distribution functions. However, the plasmas can exist in such condition for a relatively long time. Hence, this state can be considered as an effective equilibrium. Some of these regimes can lead to nonzero value of quasi-equilibrium values of the vector distribution functions. It can provide more effective application of suggested kinetic model.

AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

Author Contributions

Pavel Andreev: Investigation (equal); Writing – original draft (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

APPENDIX A: POTENTIALS OF THE ELECTROMAGNETIC FIELD IN THE MULTIPOLE APPROXIMATION

The application of the full retarding potentials for the description of the field created by particles (7) and (8) reveals in the full set of Maxwell equations found in the self-consistent field approximation (48)–(50). However, first it appears (in the self-consistent field approximation) in the kinetic equation as the integral terms and simplifies after the splitting of the two-coordinate distribution functions on the products of the corresponding one-coordinate distribution functions. These integral terms are shown in the text, but we want to specify the structure of the scalar and vector potentials of the electromagnetic field caused by different distribution functions since the Maxwell equations (48)–(50) show the combined contribution of all terms.

Therefore, the partial scalar potentials are

$$\varphi_0 = \int d\mathbf{r}' d\mathbf{p}' \int dt' G(t, t', \mathbf{r}, \mathbf{r}') f(\mathbf{r}', \mathbf{p}', t'), \tag{A1}$$

$$\varphi_D = -\partial^c_{\mathbf{r}} \int d\mathbf{r}' d\mathbf{p}' \int dt' G(t, t', \mathbf{r}, \mathbf{r}') d^c(\mathbf{r}', \mathbf{p}', t'), \tag{A2}$$

and

$$\varphi_Q = \frac{1}{2}\partial^c_{\mathbf{r}}\partial^b_{\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')q^{bc}(\mathbf{r}',\mathbf{p}',t'). \quad \text{(A3)}$$

The partial vector potentials are

$$A^a_0 = \frac{1}{c}\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')j^a(\mathbf{r}',\mathbf{p}',t'), \quad \text{(A4)}$$

$$A^a_D = -\frac{1}{c}\partial^b_{\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')J^{ab}_D(\mathbf{r}',\mathbf{p}',t'), \quad \text{(A5)}$$

and

$$A^a_Q = \frac{1}{2}\frac{1}{c}\partial^c_{\mathbf{r}}\partial^b_{\mathbf{r}}\int d\mathbf{r}'d\mathbf{p}'\int dt' G(t,t',\mathbf{r},\mathbf{r}')J^{abc}_Q(\mathbf{r}',\mathbf{p}',t'). \quad \text{(A6)}$$

These potentials correspond to the Maxwell equations presented in the text (48)–(50).

APPENDIX B: CORRELATIONLESS LIMIT OF TWO-COORDINATE DISTRIBUTION FUNCTIONS

1. Functions for Eq. (44)

The following notations for the two-coordinate distribution functions are used in Eq. (44):

$$\langle\langle v^a_i(t)\rangle\rangle \to j^a_s(\mathbf{r},\mathbf{p},t)\cdot f_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B1)}$$

$$\langle\langle v^a_j(t')\rangle\rangle \to f_s(\mathbf{r},\mathbf{p},t)\cdot j^a_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B2)}$$

$$\langle\langle \xi^b v^a_i(t)\rangle\rangle \to J^{ab}_{D,s}(\mathbf{r},\mathbf{p},t)\cdot f_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B3)}$$

$$\langle\langle \xi^b v^a_j(t')\rangle\rangle \to d^b_s(\mathbf{r},\mathbf{p},t)\cdot j^a_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B4)}$$

$$\langle\langle \xi'^b v^a_j(t')\rangle\rangle \to f_s(\mathbf{r},\mathbf{p},t)\cdot J^{ab}_{D,s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B5)}$$

$$\langle\langle \xi^b\xi^c v^a_j(t')\rangle\rangle \to q^{bc}_s(\mathbf{r},\mathbf{p},t)\cdot j^a_{s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B6)}$$

$$\langle\langle \xi'^b\xi'^c v^a_j(t')\rangle\rangle \to f_s(\mathbf{r},\mathbf{p},t)\cdot J^{abc}_{Q,s'}(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B7)}$$

and

$$\langle\langle \xi^b\xi'^c v^a_j(t')\rangle\rangle \to d^b_s(\mathbf{r},\mathbf{p},t)\cdot J^{ac}_{D,s'}(\mathbf{r}',\mathbf{p}',t'). \quad \text{(B8)}$$

2. Functions for Eq. (46)

The following notations for the two-coordinate distribution functions are used in Eq. (46):

$$\langle\langle v^b_i(t)v^g_j(t')\rangle\rangle \to j^b(\mathbf{r},\mathbf{p},t)\cdot j^g(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B9)}$$

$$\langle\langle \xi^a v^b_i(t)v^g_j(t')\rangle\rangle \to J^{ba}_D(\mathbf{r},\mathbf{p},t)\cdot j^g(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B10)}$$

$$\langle\langle \xi'^a v^b_i(t)v^g_j(t')\rangle\rangle \to j^b(\mathbf{r},\mathbf{p},t)\cdot J^{ga}_D(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B11)}$$

$$\langle\langle \xi^s\xi^l v^b_i(t)v^g_j(t')\rangle\rangle \to J^{bsl}_Q(\mathbf{r},\mathbf{p},t)\cdot j^g(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B12)}$$

$$\langle\langle \xi'^s\xi'^l v^b_i(t)v^g_j(t')\rangle\rangle \to j^b(\mathbf{r},\mathbf{p},t)\cdot J^{gsl}_Q(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B13)}$$

and

$$\langle\langle \xi^s\xi'^l v^b_i(t)v^g_j(t')\rangle\rangle \to J^{bs}_D(\mathbf{r},\mathbf{p},t)\cdot J^{gl}_D(\mathbf{r}',\mathbf{p}',t'). \quad \text{(B14)}$$

3. Functions for Eq. (62)

The following notations for the two-coordinate distribution functions are used in Eq. (62):

$$\langle\langle v^a_i(t)\rangle\rangle(\mathbf{r},\mathbf{r}',\mathbf{p},\mathbf{p}',t,t') = j^a(\mathbf{r},\mathbf{p},t)\cdot f(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B15)}$$

$$\langle\langle \xi^c v^a_i(t)\rangle\rangle = J^{ac}_D(\mathbf{r},\mathbf{p},t)\cdot f(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B16)}$$

$$\langle\langle \xi'^c v^a_i(t)\rangle\rangle = j^a(\mathbf{r},\mathbf{p},t)\cdot d^c(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B17)}$$

$$\langle\langle v^a_i(t)v^b_j(t')\rangle\rangle = j^a(\mathbf{r},\mathbf{p},t)\cdot j^b(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B18)}$$

$$\langle\langle v^a_i(t)v^c_i(t)v^g_j(t')\rangle\rangle = \langle v^a_i(t)v^c_i(t)\rangle\cdot j^g(\mathbf{r}',\mathbf{p}',t'), \quad \text{(B19)}$$

and

$$\langle\langle v^a_i(t)v^c_i(t)v^g_j(t')\xi^h\rangle\rangle = \langle v^a_i(t)v^c_i(t)\xi^h\rangle\cdot j^g(\mathbf{r}',\mathbf{p}',t'). \quad \text{(B20)}$$

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